CHAPTER 1 | *INTRODUCTION AND MATHEMATICAL CONCEPTS*

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

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1. (d) The resultant vector **R** is drawn from the tail of the first vector to the head of the last vector.

2. (c) Note from the drawing that the magnitude R of the resultant vector **R** is equal to the shortest distance between the tail of **A** and the head of **B**. Thus, R is less than the magnitude (length) of **A** plus the magnitude of **B**.

3. (a) The triangle in the drawing is a right triangle. The lengths A and B of the two sides are known, so the Pythagorean theorem can be used to determine the length R of the hypotenuse.

4. (b) The angle is found by using the inverse tangent function, $\theta = \tan^{-1}\left(\frac{4.0 \text{ km}}{3.0 \text{ km}}\right) = 53°$.

5. (b) In this drawing the vector –**C** is reversed relative to **C**, while vectors **A** and **B** are not reversed.

6. (c) In this drawing the vectors –**B** and –**C** are reversed relative to **B** and **C**, while vector **A** is not reversed.

7. (e) These vectors form a closed four-sided polygon, with the head of the fourth vector exactly meeting the tail of the first vector. Thus, the resultant vector is zero.

8. (c) When the two vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ are added by the tail-to-head method, the sum equals the vector **A**. Therefore, these vector components are the correct ones.

9. (b) The three vectors form a right triangle, so the magnitude of **A** is given by the Pythagorean theorem as $A = \sqrt{A_x^2 + A_y^2}$. If A_x and A_y double in size, then the magnitude of **A** doubles: $\sqrt{\left(2A_x\right)^2 + \left(2A_y\right)^2} = \sqrt{4A_x^2 + 4A_y^2} = 2\sqrt{A_x^2 + A_y^2} = 2A.$

10. (a) The angle θ is determined by the inverse tangent function, $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$. If A_x and A_y both become twice as large, the ratio does not change, and θ remains the same.

11. (b) The displacement vector **A** points in the $-y$ direction. Therefore, it has no scalar component along the x axis (A_x = 0 m) and its scalar component along the y axis is negative.

12. (e) The scalar components are given by $A_{x'} = -(450\text{ m})\sin 35.0° = -258\text{ m}$ and $A_{y'} = -(450\text{ m})\cos 35.0° = -369\text{ m}$.

13. (d) The distance (magnitude) traveled by each runner is the same, but the directions are different. Therefore, the two displacement vectors are not equal.

14. (c) A_x and B_x point in opposite directions, and A_y and B_y point in the same direction.

15. (d)

16. $A_y = 3.4\text{ m}$, $B_y = 3.4\text{ m}$

17. $R_x = 0\text{ m}$, $R_y = 6.8\text{ m}$

18. $R = 7.9\text{ m}$, $\theta = 21$ degrees

CHAPTER 1 *INTRODUCTION AND MATHEMATICAL CONCEPTS*

PROBLEMS

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1. ***REASONING*** We use the fact that 1 m = 3.28 ft to form the following conversion factor: (1 m)/(3.28 ft) = 1.

SOLUTION To convert ft^2 into m^2, we apply the conversion factor twice:

$$\text{Area} = \left(1330\ \cancel{\text{ft}^2}\right)\left(\frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}}\right)\left(\frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}}\right) = \boxed{124\ \text{m}^2}$$

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2. ***REASONING***

a. To convert the speed from miles per hour (mi/h) to kilometers per hour (km/h), we need to convert miles to kilometers. This conversion is achieved by using the relation 1.609 km = 1 mi (see the page facing the inside of the front cover of the text).

b. To convert the speed from miles per hour (mi/h) to meters per second (m/s), we must convert miles to meters and hours to seconds. This is accomplished by using the conversions 1 mi = 1609 m and 1 h = 3600 s.

SOLUTION a. Multiplying the speed of 34.0 mi/h by a factor of unity, (1.609 km)/(1 mi) = 1, we find the speed of the bicyclists is

$$\text{Speed} = \left(34.0\frac{\text{mi}}{\text{h}}\right)(1) = \left(34.0\frac{\cancel{\text{mi}}}{\text{h}}\right)\left(\frac{1.609\,\text{km}}{1\,\cancel{\text{mi}}}\right) = \boxed{54.7\frac{\text{km}}{\text{h}}}$$

b. Multiplying the speed of 34.0 mi/h by two factors of unity, (1609 m)/(1 mi) = 1 and (1 h)/(3600 s) = 1, the speed of the bicyclists is

$$\text{Speed} = \left(34.0\frac{\text{mi}}{\text{h}}\right)(1)(1) = \left(34.0\frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1609\,\text{m}}{1\,\cancel{\text{mi}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\,\text{s}}\right) = \boxed{15.2\frac{\text{m}}{\text{s}}}$$

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3. $\boxed{\text{SSM}}$ ***REASONING*** We use the facts that 1 mi = 5280 ft, 1 m = 3.281 ft, and 1 yd = 3 ft. With these facts we construct three conversion factors: (5280 ft)/(1 mi) = 1, (1 m)/(3.281 ft) = 1, and (3 ft)/(1 yd) = 1.

SOLUTION By multiplying by the given distance d of the fall by the appropriate conversion factors we find that

$$d = \left(6\ \cancel{\text{mi}}\right)\left(\frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}}\right) + \left(551\ \cancel{\text{yd}}\right)\left(\frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}}\right)\left(\frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}}\right) = \boxed{10\ 159\ \text{m}}$$

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4. ***REASONING*** The word "per" indicates a ratio, so "0.35 mm per day" means 0.35 mm/d, which is to be expressed as a rate in ft/century. These units differ from the given units in both length and time dimensions, so both must be converted. For length, 1 m = 10^3 mm, and 1 ft = 0.3048 m. For time, 1 year = 365.24 days, and 1 century = 100 years. Multiplying the resulting growth rate by one century gives an estimate of the total length of hair a long-lived adult could grow over his lifetime.

SOLUTION Multiply the given growth rate by the length and time conversion factors, making sure units cancel properly:

$$\text{Growth rate} = \left(0.35\frac{\cancel{\text{mm}}}{\cancel{\text{d}}}\right)\left(\frac{1\ \cancel{\text{m}}}{10^3\ \cancel{\text{mm}}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \cancel{\text{m}}}\right)\left(\frac{365.24\ \cancel{\text{d}}}{1\ \cancel{\text{y}}}\right)\left(\frac{100\ \cancel{\text{y}}}{\text{century}}\right) = \boxed{42\ \text{ft/century}}$$

---

5. ***REASONING*** In order to calculate d, the units of a and b must be, respectively, cubed and squared along with their numerical values, then combined algebraically with each other and the units of c. Ignoring the values and working first with the units alone, we have

$$d = \frac{a^3}{cb^2} \rightarrow \frac{(\text{m})^3}{(\text{m/s})(\text{s})^2} = \frac{\text{m}^{\cancel{3}\,2}}{\left(\cancel{\text{m}}/\cancel{\text{s}}\right)\cdot \text{s}^{\cancel{2}\,1}} = \frac{\text{m}^2}{\text{s}}$$

Therefore, the units of d are m^2/s.

SOLUTION With the units known, the numerical value may be calculated:

$$d = \frac{(9.7)^3}{(69)(4.2)^2}\ \text{m}^2/\text{s} = \boxed{0.75\ \text{m}^2/\text{s}}$$

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6. ***REASONING*** The dimensions of the variables v, x, and t are known, and the numerical factor 3 is dimensionless. Therefore, we can solve the equation for z and then substitute the known dimensions. The dimensions $[\text{L}]$ and $[\text{T}]$ can be treated as algebraic quantities to determine the dimensions of the variable z.

SOLUTION Since $v = \frac{1}{3}zxt^2$, it follows that $z = \dfrac{3v}{xt^2}$. We know the following dimensions: $v = [\mathrm{L}]/[\mathrm{T}]$, $x = [\mathrm{L}]$, and $t = [\mathrm{T}]$. Since the factor 3 is dimensionless, z has the dimensions of

$$\frac{v}{xt^2} = \frac{[\mathrm{L}]/[\mathrm{T}]}{[\mathrm{L}][\mathrm{T}]^2} = \boxed{\frac{1}{[\mathrm{T}]^3}}$$

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7. SSM ***REASONING*** This problem involves using unit conversions to determine the number of magnums in one jeroboam. The necessary relationships are

$$1.0 \text{ magnum} = 1.5 \text{ liters}$$
$$1.0 \text{ jeroboam} = 0.792 \text{ U. S. gallons}$$
$$1.00 \text{ U. S. gallon} = 3.785 \times 10^{-3} \text{ m}^3 = 3.785 \text{ liters}$$

These relationships may be used to construct the appropriate conversion factors.

SOLUTION By multiplying one jeroboam by the appropriate conversion factors we can determine the number of magnums in a jeroboam as shown below:

$$(1.0 \ \cancel{\text{jeroboam}})\left(\frac{0.792 \ \cancel{\text{gallons}}}{1.0 \ \cancel{\text{jeroboam}}}\right)\left(\frac{3.785 \ \cancel{\text{liters}}}{1.0 \ \cancel{\text{gallon}}}\right)\left(\frac{1.0 \text{ magnum}}{1.5 \ \cancel{\text{liters}}}\right) = \boxed{2.0 \text{ magnums}}$$

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8. ***REASONING*** By multiplying the quantity 1.78×10^{-3} kg/(s·m) by the appropriate conversions factors, we can convert the quantity to units of poise (P). These conversion factors are obtainable from the following relationships between the various units:

$$1 \text{ kg} = 1.00 \times 10^3 \text{ g}$$
$$1 \text{ m} = 1.00 \times 10^2 \text{ cm}$$
$$1 \text{ P} = 1 \text{ g}/(\text{s} \cdot \text{cm})$$

SOLUTION The conversion from the unit kg/(s·m) to the unit P proceeds as follows:

$$\left(1.78 \times 10^{-3} \ \frac{\cancel{\text{kg}}}{\cancel{\text{s}} \cdot \cancel{\text{m}}}\right)\left(\frac{1.00 \times 10^3 \ \cancel{\text{g}}}{1 \ \cancel{\text{kg}}}\right)\left(\frac{1 \ \cancel{\text{m}}}{1.00 \times 10^2 \ \cancel{\text{cm}}}\right)\left[\frac{1 \text{ P}}{1 \ \cancel{\text{g}}/(\cancel{\text{s}} \cdot \cancel{\text{cm}})}\right] = \boxed{1.78 \times 10^{-2} \text{ P}}$$

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9. ***REASONING*** Multiplying an equation by a factor of 1 does not alter the equation; this is the basis of our solution. We will use factors of 1 in the following forms:

$$\frac{1\text{ gal}}{128\text{ oz}} = 1\text{, since 1 gal = 128 oz}$$

$$\frac{3.785\times10^{-3}\text{ m}^3}{1\text{ gal}} = 1\text{, since } 3.785 \times 10^{-3}\text{ m}^3 = 1\text{ gal}$$

$$\frac{1\text{ mL}}{10^{-6}\text{m}^3} = 1\text{, since 1 mL} = 10^{-6}\text{ m}^3$$

SOLUTION The starting point for our solution is the fact that

$$\text{Volume = 1 oz}$$

Multiplying this equation on the right by factors of 1 does not alter the equation, so it follows that

$$\text{Volume } = (1\text{ oz})(1)(1)(1) = (1\ \cancel{\text{oz}})\left(\frac{1\ \cancel{\text{gal}}}{128\ \cancel{\text{oz}}}\right)\left(\frac{3.785\times10^{-3}\ \cancel{\text{m}^3}}{1\ \cancel{\text{gal}}}\right)\left(\frac{1\text{ mL}}{10^{-6}\ \cancel{\text{m}^3}}\right) = \boxed{29.6\text{ mL}}$$

Note that all the units on the right, except one, are eliminated algebraically, leaving only the desired units of milliliters (mL).

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10. ***REASONING*** To convert from gallons to cubic meters, use the equivalence 1 U.S. gal = 3.785×10^{-3} m^3. To find the thickness of the painted layer, we use the fact that the paint's volume is the same, whether in the can or painted on the wall. The layer of paint on the wall can be thought of as a very thin "box" with a volume given by the product of the surface area (the "box top") and the thickness of the layer. Therefore, its thickness is the ratio of the volume to the painted surface area: Thickness = Volume/Area. That is, the larger the area it's spread over, the thinner the layer of paint.

SOLUTION
a. The conversion is

$$\left(0.67\ \cancel{\text{U.S. gallons}}\right)\left(\frac{3.785\times10^{-3}\text{ m}^3}{\cancel{\text{U.S. gallons}}}\right) = \boxed{2.5\times10^{-3}\text{ m}^3}$$

b. The thickness is the volume found in (a) divided by the area,

$$\text{Thickness} = \frac{\text{Volume}}{\text{Area}} = \frac{2.5\times10^{-3}\text{ m}^3}{13\text{ m}^2} = \boxed{1.9\times10^{-4}\text{ m}}$$

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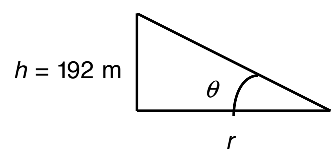
11. SSM ***REASONING*** The dimension of the spring constant k can be determined by first solving the equation $T = 2\pi\sqrt{m/k}$ for k in terms of the time T and the mass m. Then, the dimensions of T and m can be substituted into this expression to yield the dimension of k.

SOLUTION Algebraically solving the expression above for k gives $k = 4\pi^2 m/T^2$. The term $4\pi^2$ is a numerical factor that does not have a dimension, so it can be ignored in this analysis. Since the dimension for mass is [M] and that for time is [T], the dimension of k is

$$\text{Dimension of } k = \boxed{\frac{[\text{M}]}{[\text{T}]^2}}$$

---

12. ***REASONING AND SOLUTION*** The following figure (*not* drawn to scale) shows the geometry of the situation, when the observer is a distance r from the base of the arch. The angle θ is related to r and h by $\tan\theta = h/r$. Solving for r, we find



$$r = \frac{h}{\tan\theta} = \frac{192 \text{ m}}{\tan 2.0°} = 5.5 \times 10^3 \text{ m} = \boxed{5.5 \text{ km}}$$

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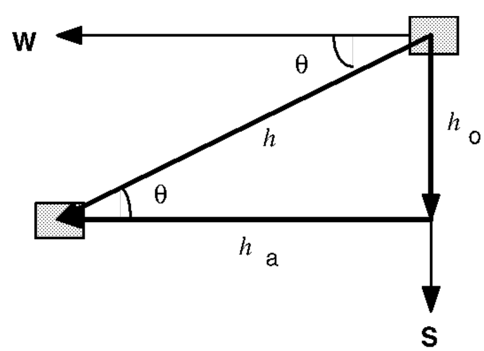
13. SSM ***REASONING*** The shortest distance between the two towns is along the line that joins them. This distance, h, is the hypotenuse of a right triangle whose other sides are h_o = 35.0 km and h_a = 72.0 km, as shown in the figure below.

SOLUTION The angle θ is given by $\tan\theta = h_o/h_a$ so that



$$\theta = \tan^{-1}\left(\frac{35.0 \text{ km}}{72.0 \text{ km}}\right) = \boxed{25.9° \text{ S of W}}$$

We can then use the Pythagorean theorem to find h.

$$h = \sqrt{h_o^2 + h_a^2} = \sqrt{(35.0 \text{ km})^2 + (72.0 \text{ km})^2} = \boxed{80.1 \text{ km}}$$

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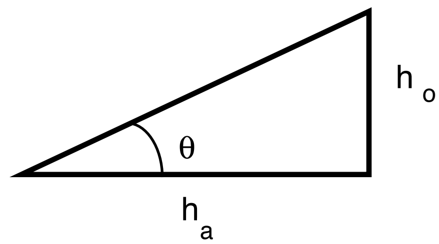
14. ***REASONING*** The drawing shows a schematic representation of the hill. We know that the hill rises 12.0 m vertically for every 100.0 m of distance in the horizontal direction, so that $h_o = 12.0$ m and $h_a = 100.0$ m. Moreover, according to Equation 1.3, the tangent function is $\tan\theta = h_o / h_a$. Thus, we can use the inverse tangent function to determine the angle θ.



SOLUTION With the aid of the inverse tangent function (see Equation 1.6) we find that

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{12.0\text{ m}}{100.0\text{ m}}\right) = \boxed{6.84°}$$

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15. ***REASONING*** Using the Pythagorean theorem (Equation 1.7), we find that the relation between the length D of the diagonal of the square (which is also the diameter of the circle) and the length L of one side of the square is $D = \sqrt{L^2 + L^2} = \sqrt{2}L$.

SOLUTION Using the above relation, we have

$$D = \sqrt{2}L \quad \text{or} \quad L = \frac{D}{\sqrt{2}} = \frac{0.35\text{ m}}{\sqrt{2}} = \boxed{0.25\text{ m}}$$

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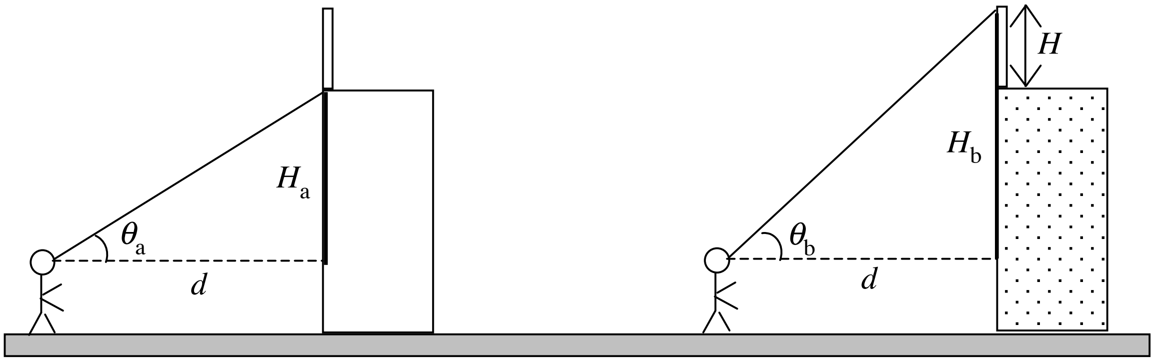
16. ***REASONING*** In both parts of the drawing the line of sight, the horizontal dashed line, and the vertical form a right triangle. The angles $\theta_a = 35.0°$ and $\theta_b = 38.0°$ at which the person's line of sight rises above the horizontal are known, as is the horizontal distance $d = 85.0$ m from the building. The unknown vertical sides of the right triangles correspond, respectively, to the heights H_a and H_b of the bottom and top of the antenna relative to the person's eyes. The antenna's height H is the *difference* between H_b and H_a: $H = H_b - H_a$. The horizontal side d of the triangle is adjacent to the angles θ_a and θ_b, while the vertical sides H_a and H_b are opposite these angles. Thus, in either triangle, the angle θ is related to the horizontal and vertical sides by Equation 1.3 $\left(\tan\theta = \frac{h_o}{h_a}\right)$:

$$\tan\theta_a = \frac{H_a}{d} \tag{1}$$

$$\tan\theta_b = \frac{H_b}{d} \tag{2}$$



(*a*) (*b*)

SOLUTION Solving Equations (1) and (2) for the heights of the bottom and top of the antenna relative to the person's eyes, we find that

$$H_a = d\tan\theta_a \qquad \text{and} \qquad H_b = d\tan\theta_b$$

The height of the antenna is the difference between these two values:

$$H = H_b - H_a = d\tan\theta_b - d\tan\theta_a = d\left(\tan\theta_b - \tan\theta_a\right)$$

$$H = (85.0\text{ m})\left(\tan 38.0^\circ - \tan 35.0^\circ\right) = \boxed{6.9\text{ m}}$$

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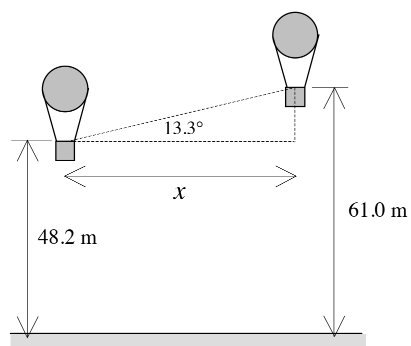
17. ***REASONING*** The drawing shows the heights of the two balloonists and the horizontal distance x between them. Also shown in dashed lines is a right triangle, one angle of which is 13.3°. Note that the side adjacent to the 13.3° angle is the horizontal distance x, while the side opposite the angle is the distance between the two heights, 61.0 m − 48.2 m. Since we know the angle and the length of one side of the right triangle, we can use trigonometry to find the length of the other side.



SOLUTION The definition of the tangent function, Equation 1.3, can be used to find the horizontal distance x, since the angle and the length of the opposite side are known:

$$\tan 13.3^\circ = \frac{\text{length of opposite side}}{\text{length of adjacent side } (= x)}$$

Solving for x gives

$$x = \frac{\text{length of opposite side}}{\tan 13.3^\circ} = \frac{61.0\text{ m} - 48.2\text{ m}}{\tan 13.3^\circ} = \boxed{54.1\text{ m}}$$

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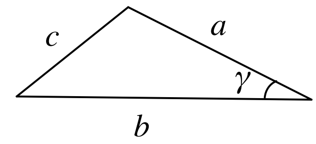
18. ***REASONING*** As given in Appendix E, the law of cosines is



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

where c is the side opposite angle γ, and a and b are the other two sides. Solving for γ, we have

$$\gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

SOLUTION For c = 95 cm, a = 150 cm, and b = 190 cm

$$\gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) = \cos^{-1}\left[\frac{(150\text{ cm})^2 + (190)^2 - (95\text{ cm})^2}{2(150\text{ cm})(190\text{ cm})}\right] = 30°$$

Thus, the angle opposite the side of length 95 cm is $\boxed{30°}$.

Similarly, for c = 150 cm, a = 95 cm, and b =190 cm, we find that the angle opposite the side of length 150 cm is $\boxed{51°}$.

Finally, for c = 190 cm, a = 150 cm, and b = 95 cm, we find that the angle opposite the side of length 190 cm is $\boxed{99°}$.

As a check on these calculations, we note that $30° + 51° + 99° = 180°$, which must be the case for the sum of the three angles in a triangle.

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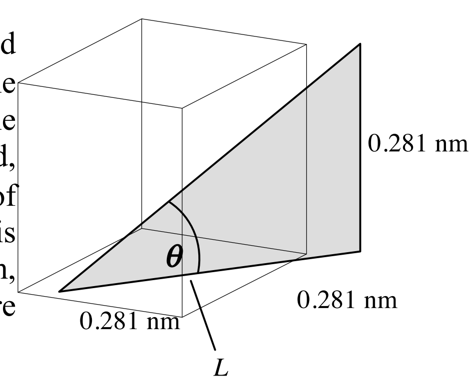
19. ***REASONING*** Note from the drawing that the shaded right triangle contains the angle θ, the side opposite the angle (length = 0.281 nm), and the side adjacent to the angle (length = L). If the length L can be determined, we can use trigonometry to find θ. The bottom face of the cube is a square whose diagonal has a length L. This length can be found from the Pythagorean theorem, since the lengths of the two sides of the square are known.



SOLUTION The angle can be obtained from the inverse tangent function, Equation 1.6, as $\theta = \tan^{-1}\left[(0.281\text{ nm})/L\right]$. Since L is the length of the hypotenuse of a right triangle whose sides have lengths of 0.281 nm, its value can be determined from the Pythagorean theorem:

$$L = \sqrt{(0.281\ \text{nm})^2 + (0.281\ \text{nm})^2} = 0.397\ \text{nm}$$

Thus, the angle is

$$\theta = \tan^{-1}\left(\frac{0.281\ \text{nm}}{L}\right) = \tan^{-1}\left(\frac{0.281\ \text{nm}}{0.397\ \text{nm}}\right) = \boxed{35.3^\circ}$$

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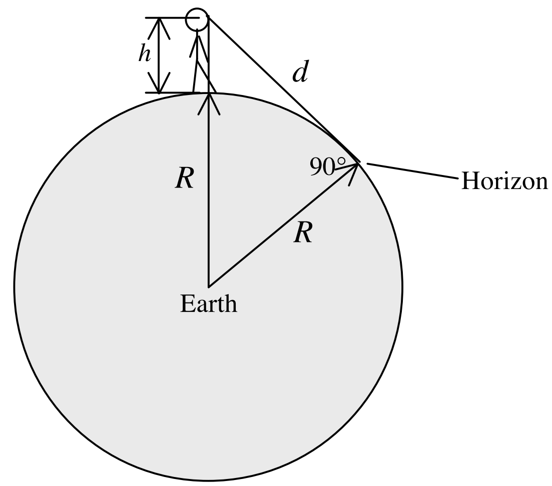
20. ***REASONING***

a. The drawing shows the person standing on the earth and looking at the horizon. Notice the right triangle, the sides of which are R, the radius of the earth, and d, the distance from the person's eyes to the horizon. The length of the hypotenuse is $R + h$, where h is the height of the person's eyes above the water. Since we know the lengths of two sides of the triangle, the Pythagorean theorem can be employed to find the length of the third side.



b. To convert the distance from meters to miles, we use the relation 1609 m = 1 mi (see the page facing the inside of the front cover of the text).

SOLUTION

a. The Pythagorean theorem (Equation 1.7) states that the square of the hypotenuse is equal to the sum of the squares of the sides, or $(R+h)^2 = d^2 + R^2$. Solving this equation for d yields

$$d = \sqrt{(R+h)^2 - R^2} = \sqrt{R^2 + 2Rh + h^2 - R^2}$$

$$= \sqrt{2Rh + h^2} = \sqrt{2\left(6.38\times10^6\ \text{m}\right)(1.6\ \text{m}) + (1.6\ \text{m})^2} = \boxed{4500\ \text{m}}$$

b. Multiplying the distance of 4500 m by a factor of unity, (1 mi)/(1609 m) = 1, the distance (in miles) from the person's eyes to the horizon is

$$d = (4500\ \text{m})(1) = \left(4500\ \cancel{\text{m}}\right)\left(\frac{1\ \text{mi}}{1609\ \cancel{\text{m}}}\right) = \boxed{2.8\ \text{mi}}$$

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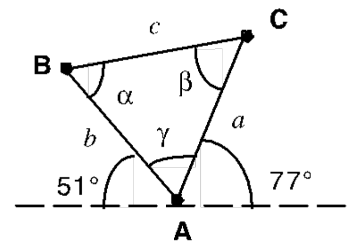
21. SSM ***REASONING*** The drawing at the right shows the location of each deer A, B, and C. From the problem statement it follows that



$$b = 62 \text{ m}$$

$$c = 95 \text{ m}$$

$$\gamma = 180° - 51° - 77° = 52°$$

Applying the law of cosines (given in Appendix E) to the geometry in the figure, we have

$$a^2 - 2ab\cos\gamma + (b^2 - c^2) = 0$$

which is an expression that is quadratic in a. It can be simplified to $Aa^2 + Ba + C = 0$, with

$$A = 1$$

$$B = -2b\cos\gamma = -2(62 \text{ m})\cos 52° = -76 \text{ m}$$

$$C = (b^2 - c^2) = (62 \text{ m})^2 - (95 \text{ m})^2 = -5181 \text{ m}^2$$

This quadratic equation can be solved for the desired quantity a.

SOLUTION Suppressing units, we obtain from the quadratic formula

$$a = \frac{-(-76) \pm \sqrt{(-76)^2 - 4(1)(-5181)}}{2(1)} = 1.2 \times 10^2 \text{ m} \quad \text{and} \quad -43 \text{ m}$$

Discarding the negative root, which has no physical significance, we conclude that the distance between deer A and C is $\boxed{1.2 \times 10^2 \text{ m}}$.

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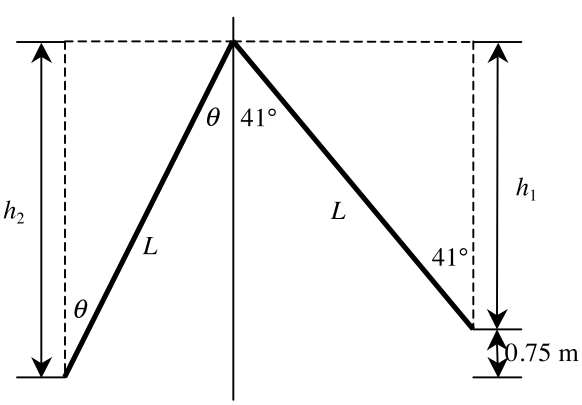
22. ***REASONING*** The trapeze cord is $L = 8.0$ m long, so that the trapeze is initially $h_1 = L\cos 41°$ meters below the support. At the instant he releases the trapeze, it is $h_2 = L\cos\theta$ meters below the support. The difference in the heights is $d = h_2 - h_1 = 0.75$ m. Given that the trapeze is released at a lower elevation than the platform, we expect to find $\theta < 41°$.



SOLUTION Putting the above relationships together, we have

$$d = h_2 - h_1 = L\cos\theta - L\cos 41^\circ \qquad \text{or} \qquad d + L\cos 41^\circ = L\cos\theta$$

$$\cos\theta = \frac{d}{L} + \cos 41^\circ$$

$$\theta = \cos^{-1}\left(\frac{d}{L} + \cos 41^\circ\right) = \cos^{-1}\left(\frac{0.75 \text{ m}}{8.0 \text{ m}} + \cos 41^\circ\right) = \boxed{32^\circ}$$

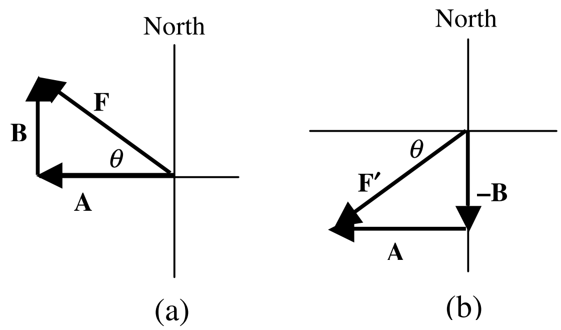
---

23. SSM ***REASONING***

a. Since the two force vectors **A** and **B** have directions due west and due north, they are perpendicular. Therefore, the resultant vector **F = A + B** has a magnitude given by the Pythagorean theorem: $F^2 = A^2 + B^2$. Knowing the magnitudes of **A** and **B**, we can calculate the magnitude of **F**. The direction of the resultant can be obtained using trigonometry.

b. For the vector **F′ = A – B** we note that the subtraction can be regarded as an addition in the following sense: **F′ = A + (–B)**. The vector **–B** points due south, opposite the vector **B**, so the two vectors are once again perpendicular and the magnitude of **F′** again is given by the Pythagorean theorem. The direction again can be obtained using trigonometry.

SOLUTION a. The drawing shows the two vectors and the resultant vector. According to the Pythagorean theorem, we have



$$F^2 = A^2 + B^2$$

$$F = \sqrt{A^2 + B^2}$$

$$F = \sqrt{(445 \text{ N})^2 + (325 \text{ N})^2}$$

$$= \boxed{551 \text{ N}}$$

Using trigonometry, we can see that the direction of the resultant is

$$\tan\theta = \frac{B}{A} \qquad \text{or} \qquad \theta = \tan^{-1}\left(\frac{325 \text{ N}}{445 \text{ N}}\right) = \boxed{36.1^\circ \text{ north of west}}$$

b. Referring to the drawing and following the same procedure as in part a, we find

$$F'^2 = A^2 + (-B)^2 \quad \text{or} \quad F' = \sqrt{A^2 + (-B)^2} = \sqrt{(445\text{ N})^2 + (-325\text{ N})^2} = \boxed{551\text{ N}}$$

$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{325\text{ N}}{445\text{ N}}\right) = \boxed{36.1^\circ \text{ south of west}}$$

---

24. ***REASONING*** Since the initial force and the resultant force point along the east/west line, the second force must also point along the east/west line. The direction of the second force is not specified; it could point either due east or due west, so there are two answers. We use "N" to denote the units of the forces, which are specified in newtons.

SOLUTION If the second force points $\boxed{\text{due east}}$**,** both forces point in the same direction and the magnitude of the resultant force is the sum of the two magnitudes: $F_1 + F_2 = F_R$. Therefore,

$$F_2 = F_R - F_1 = 400\text{ N} - 200\text{ N} = \boxed{200\text{ N}}$$

If the second force points $\boxed{\text{due west}}$, the two forces point in opposite directions, and the magnitude of the resultant force is the difference of the two magnitudes: $F_2 - F_1 = F_R$. Therefore,

$$F_2 = F_R + F_1 = 400\text{ N} + 200\text{ N} = \boxed{600\text{ N}}$$

---

25. $\boxed{\text{SSM}}$ ***REASONING*** For convenience, we can assign due east to be the positive direction and due west to be the negative direction. Since all the vectors point along the same east-west line, the vectors can be added just like the usual algebraic addition of positive and negative scalars. We will carry out the addition for all of the possible choices for the two vectors and identify the resultants with the smallest and largest magnitudes.

SOLUTION There are six possible choices for the two vectors, leading to the following resultant vectors:

$$\mathbf{F_1} + \mathbf{F_2} = 50.0\text{ newtons} + 10.0\text{ newtons} = +60.0\text{ newtons} = 60.0\text{ newtons, due east}$$

$$\mathbf{F_1} + \mathbf{F_3} = 50.0\text{ newtons} - 40.0\text{ newtons} = +10.0\text{ newtons} = 10.0\text{ newtons, due east}$$

$$\mathbf{F_1} + \mathbf{F_4} = 50.0\text{ newtons} - 30.0\text{ newtons} = +20.0\text{ newtons} = 20.0\text{ newtons, due east}$$

$$\mathbf{F_2} + \mathbf{F_3} = 10.0\text{ newtons} - 40.0\text{ newtons} = -30.0\text{ newtons} = 30.0\text{ newtons, due west}$$

$$\mathbf{F_2} + \mathbf{F_4} = 10.0\text{ newtons} - 30.0\text{ newtons} = -20.0\text{ newtons} = 20.0\text{ newtons, due west}$$

$$\mathbf{F_3} + \mathbf{F_4} = -40.0\text{ newtons} - 30.0\text{ newtons} = -70.0\text{ newtons} = 70.0\text{ newtons, due west}$$

The resultant vector with the smallest magnitude is $\boxed{\mathbf{F_1} + \mathbf{F_3} = 10.0 \text{ newtons, due east}}$.

The resultant vector with the largest magnitude is $\boxed{\mathbf{F_3} + \mathbf{F_4} = 70.0 \text{ newtons, due west}}$.

---

26. ***REASONING*** The Pythagorean theorem (Equation 1.7) can be used to find the magnitude of the resultant vector, and trigonometry can be employed to determine its direction.

a. Arranging the vectors in tail-to-head fashion, we can see that the vector **A** gives the resultant a westerly direction and vector **B** gives the resultant a southerly direction. Therefore, the resultant **A** + **B** points south of west.

b. Arranging the vectors in tail-to-head fashion, we can see that the vector **A** gives the resultant a westerly direction and vector **–B** gives the resultant a northerly direction. Therefore, the resultant **A** + **(–B)** points north of west.

SOLUTION Using the Pythagorean theorem and trigonometry, we obtain the following results:

a.
$$\text{Magnitude of } \mathbf{A} + \mathbf{B} = \sqrt{(63 \text{ units})^2 + (63 \text{ units})^2} = \boxed{89 \text{ units}}$$

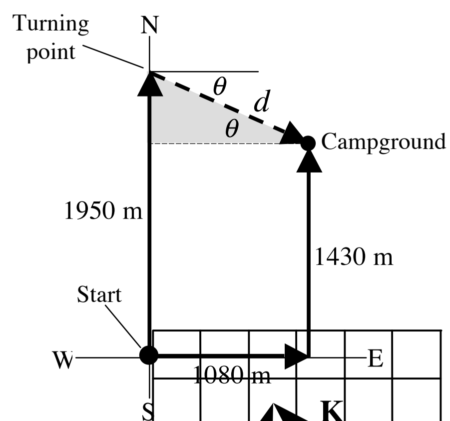
$$\theta = \tan^{-1}\left(\frac{63 \text{ units}}{63 \text{ units}}\right) = \boxed{45^\circ \text{ south of west}}$$

b.
$$\text{Magnitude of } \mathbf{A} - \mathbf{B} = \sqrt{(63 \text{ units})^2 + (63 \text{ units})^2} = \boxed{89 \text{ units}}$$

$$\theta = \tan^{-1}\left(\frac{63 \text{ units}}{63 \text{ units}}\right) = \boxed{45^\circ \text{ north of west}}$$

–

27. ***REASONING*** At the turning point, the distance to the campground is labeled d in the drawing. Note that d is the length of the hypotenuse of a right triangle. Since we know the lengths of the other two sides of the triangle, the Pythagorean theorem can be used to find d. The direction that cyclist #2 must head during the last part of the trip is given by the angle θ. It can be determined by using the inverse tangent function.



SOLUTION
a. The two sides of the triangle have lengths of 1080 m and 520 m (1950 m − 1430 m = 520 m). The length d of the hypotenuse can be determined from the Pythagorean theorem, Equation (1.7), as

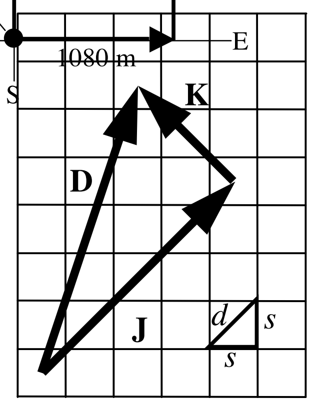
$$d = \sqrt{(1080\text{ m})^2 + (520\text{ m})^2} = \boxed{1200\text{ m}}$$

b. Since the lengths of the sides opposite and adjacent to the angle θ are known, the inverse tangent function (Equation 1.6) can be used to find θ.

$$\theta = \tan^{-1}\left(\frac{520\text{ m}}{1080\text{ m}}\right) = \boxed{26^\circ\text{ south of east}}$$

---

28. ***REASONING*** The triple jump consists of a double jump in one direction, followed by a perpendicular single jump, which we can represent with displacement vectors **J** and **K** (see the drawing). These two perpendicular vectors form a right triangle with their resultant **D** = **J** + **K**, which is the displacement of the colored checker. In order to find the magnitude D of the displacement, we first need to find the magnitudes J and K of the double jump and the single jump. As the three sides of a right triangle, J, K, and D (the hypotenuse) are related to one another by the Pythagorean theorem (Equation 1.7) The double jump moves the colored checker a straight-line distance equal to the length of four square's diagonals d, and the single jump moves a length equal to two square's diagonals. Therefore,

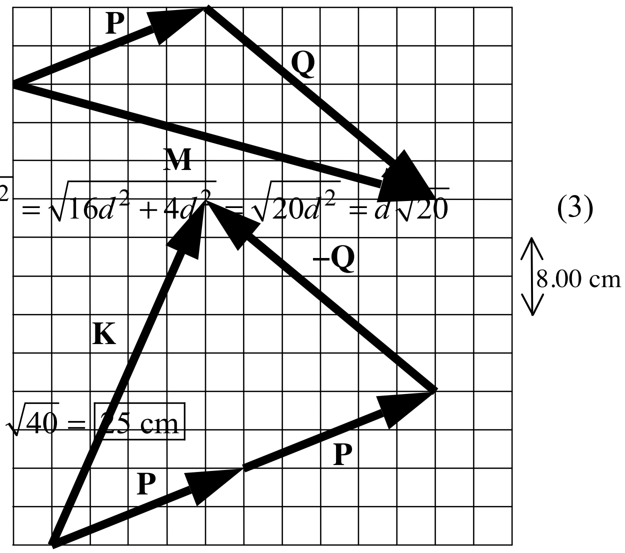


$$J = 4d \qquad \text{and} \qquad K = 2d \tag{1}$$

Let the length of a square's side be s. Any two adjacent sides of a square form a right triangle with the square's diagonal (see the drawing). The Pythagorean theorem gives the diagonal length d in terms of the side length s:

$$d = \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2} \tag{2}$$

SOLUTION First, we apply the Pythagorean theorem to the right triangle formed by the three displacement vectors, using Equations (1) for J and K:



$$D = \sqrt{J^2 + K^2} = \sqrt{(4d)^2 + (2d)^2} = \sqrt{16d^2 + 4d^2} = \sqrt{20d^2} = d\sqrt{20} \qquad (3)$$

Substituting Equation (2) into Equation (3) gives

$$D = d\sqrt{20} = \left(s\sqrt{2}\right)\sqrt{20} = s\sqrt{40} = (4.0\text{ cm})\sqrt{40} = \boxed{25\text{ cm}}$$

---

29. ***REASONING*** Both **P** and **Q** and the vector sums **K** and **M** can be drawn with correct magnitudes and directions by counting grid squares. To add vectors, place them tail-to-head and draw the resultant vector from the tail of the first vector to the head of the last. The vector 2**P** is equivalent to **P** + **P**, and −**Q** is a vector that has the same magnitude as **Q**, except it is directed in the opposite direction.

The vector **M** runs 11 squares horizontally and 3 squares vertically, and the vector **K** runs 4 squares horizontally and 9 squares vertically. These distances can be converted from grid squares to centimeters with the grid scale: 1 square = 4.00 cm. Once the distances are calculated in centimeters, the Pythagorean theorem (Equation 1.7) will give the magnitudes of the vectors.

SOLUTION

a. The vector **M** = **P** + **Q** runs 11 squares horizontally and 3 squares vertically, and these distances are equivalent to, respectively, $\left(4.00\ \dfrac{\text{cm}}{\text{square}}\right)(11\text{ squares}) = 44.0\text{ cm}$ and $\left(4.00\ \dfrac{\text{cm}}{\text{square}}\right)(3\text{ squares}) = 12.0\text{ cm}$. Thus, the magnitude of **M** is

$$M = \sqrt{(44.0\text{ cm})^2 + (12.0\text{ cm})^2} = \boxed{45.6\text{ cm}}$$

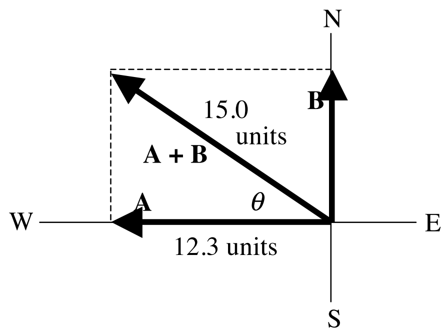
b. Similarly, the lengths of the horizontal and vertical distances of **K** = 2**P** − **Q** are 4 horizontal squares and 9 vertical squares, or 16.0 cm and 36.0 cm, respectively. The magnitude of **K** is then

$$K = \sqrt{(16.0\text{ cm})^2 + (36.0\text{ cm})^2} = \boxed{39.4\text{ cm}}$$

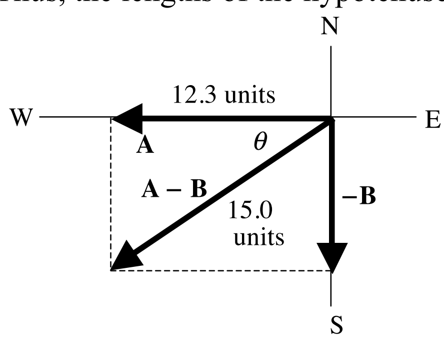
---

30. ***REASONING***

a. and b. The following drawing shows the two vectors **A** and **B**, as well as the resultant vector **A** + **B**. The three vectors form a right triangle, of which two of the sides are known. We can employ the Pythagorean theorem, Equation 1.7, to find the length of the third side. The angle θ in the drawing can be determined by using the inverse cosine function, Equation 1.5, since the side adjacent to θ and the length of the hypotenuse are known.



c. and d. The following drawing shows the vectors **A** and **–B**, as well as the resultant vector **A – B.** The three vectors form a right triangle, which is identical to the previous drawing, except for orientation. Thus, the lengths of the hypotenuses and the angles are equal.



SOLUTION

a. Let **R** = **A** + **B**. The Pythagorean theorem (Equation 1.7) states that the square of the hypotenuse is equal to the sum of the squares of the sides, so that $R^2 = A^2 + B^2$. Solving for B yields

$$B = \sqrt{R^2 - A^2} = \sqrt{(15.0\text{ units})^2 - (12.3\text{ units})^2} = \boxed{8.6\text{ units}}$$

b. The angle θ can be found from the inverse cosine function, Equation 1.5:

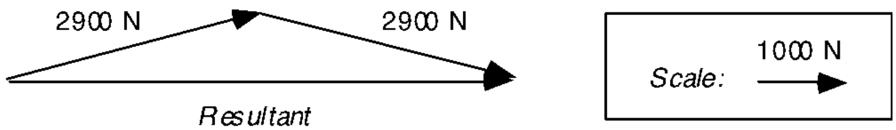
$$\theta = \cos^{-1}\left(\frac{12.3\text{ units}}{15.0\text{ units}}\right) = \boxed{34.9^\circ\text{ north of west}}$$

c. Except for orientation, the triangles in the two drawings are the same. Thus, the value for B is the same as that determined in part (a) above: $B = \boxed{8.6\text{ units}}$

d. The angle θ is the same as that found in part (a), except the resultant vector points south of west, rather than north of west: $\theta = \boxed{34.9° \text{ south of west}}$

---

31. SSM ***REASONING AND SOLUTION*** The single force needed to produce the same effect is equal to the resultant of the forces provided by the two ropes. The following figure shows the force vectors drawn to scale and arranged tail to head. The magnitude and direction of the resultant can be found by direct measurement using the scale factor shown in the figure.



a. From the figure, the magnitude of the resultant is $\boxed{5600 \text{ N}}$.

b. The single rope should be directed $\boxed{\text{along the dashed line}}$ in the text drawing.

---

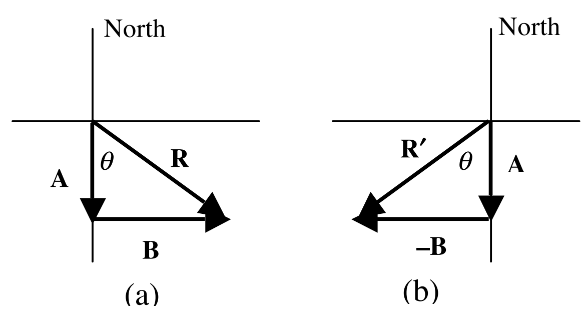
32. ***REASONING*** a. Since the two displacement vectors **A** and **B** have directions due south and due east, they are perpendicular. Therefore, the resultant vector **R = A + B** has a magnitude given by the Pythagorean theorem: $R^2 = A^2 + B^2$. Knowing the magnitudes of **R** and **A**, we can calculate the magnitude of **B**. The direction of the resultant can be obtained using trigonometry.

b. For the vector **R′** = **A – B** we note that the subtraction can be regarded as an addition in the following sense: **R′ = A + (–B)**. The vector **–B** points due west, opposite the vector **B**, so the two vectors are once again perpendicular and the magnitude of **R′** again is given by the Pythagorean theorem. The direction again can be obtained using trigonometry.

SOLUTION a. The drawing shows the two vectors and the resultant vector. According to the Pythagorean theorem, we have

$$R^2 = A^2 + B^2 \quad \text{or} \quad B = \sqrt{R^2 - A^2}$$

$$B = \sqrt{(3.75 \text{ km})^2 - (2.50 \text{ km})^2} = \boxed{2.8 \text{ km}}$$



Using trigonometry, we can see that the direction of the resultant is

$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{2.8 \text{ km}}{2.50 \text{ km}}\right) = \boxed{48° \text{ east of south}}$$

b. Referring to the drawing and following the same procedure as in part a, we find

$$R'^2 = A^2 + (-B)^2 \quad \text{or} \quad B = \sqrt{R'^2 - A^2} = \sqrt{(3.75\text{ km})^2 - (2.50\text{ km})^2} = \boxed{2.8\text{ km}}$$

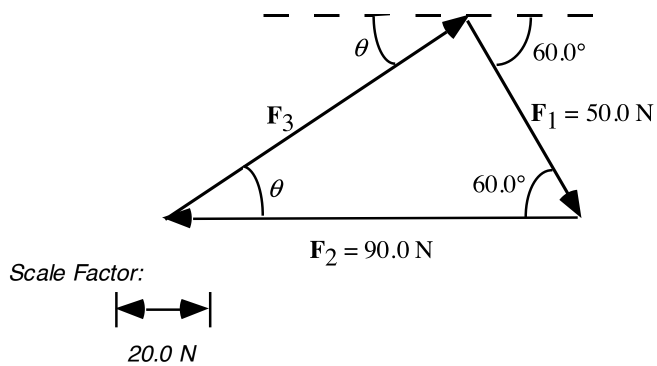
$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{2.8\text{ km}}{2.50\text{km}}\right) = \boxed{48° \text{ west of south}}$$

---

33. ***REASONING AND SOLUTION*** The following figure is a scale diagram of the forces drawn tail-to-head. The scale factor is shown in the figure. The head of $\mathbf{F}_3$ touches the tail of $\mathbf{F}_1$, because the resultant of the three forces is zero.

a. From the figure, $\mathbf{F}_3$ must have a magnitude of $\boxed{78\text{ N}}$ if the resultant force acting on the ball is zero.

b. Measurement with a protractor indicates that the angle $\boxed{\theta = 34°}$.



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34. ***REASONING*** The magnitude of the *x*-component of the force vector is the product of the magnitude of the force times the cosine of the angle between the vector and the *x* axis. Since the *x*-component points in the +*x* direction, it is positive. Likewise, the magnitude of the *y* component of the force vector is the product of the magnitude of the force times the sine of the angle between the vector and the *x* axis. Since the vector points 36.0º below the positive *x* axis, the *y* component of the vector points in the −*y* direction; thus, a minus sign must be assigned to the *y*-component to indicate this direction.

SOLUTION The *x* and *y* scalar components are

a. $F_x = (575\text{ newtons})\cos 36.0° = \boxed{465\text{ newtons}}$

b. $F_y = -(575\text{ newtons})\sin 36.0° = \boxed{-338\text{ newtons}}$

---

35. **SSM** ***REASONING AND SOLUTION*** In order to determine which vector has the largest *x* and *y* components, we calculate the magnitude of the *x* and *y* components explicitly and compare them. In the calculations, the symbol u denotes the units of the vectors.

$A_x = (100.0\text{ u})\cos 90.0° = 0.00\text{ u}$ $\qquad$ $A_y = (100.0\text{ u})\sin 90.0° = 1.00\times 10^2\text{ u}$

$B_x = (200.0\text{ u})\cos 60.0° = 1.00\times 10^2\text{ u}$ $\qquad$ $B_y = (200.0\text{ u})\sin 60.0° = 173\text{ u}$

$C_x = (150.0\text{ u})\cos 0.00° = 150.0\text{ u}$ $\qquad$ $C_y = (150.0\text{ u})\sin 0.00° = 0.00\text{ u}$

a. $\boxed{\textbf{C}\text{ has the largest } x \text{ component.}}$

b. $\boxed{\textbf{B}\text{ has the largest } y \text{ component.}}$

---

36. ***REASONING*** The triangle in the drawing is a right triangle. We know one of its angles is 30.0°, and the length of the hypotenuse is 8.6 m. Therefore, the sine and cosine functions can be used to find the magnitudes of $\mathbf{A_x}$ and $\mathbf{A_y}$. The directions of these vectors can be found by examining the diagram.

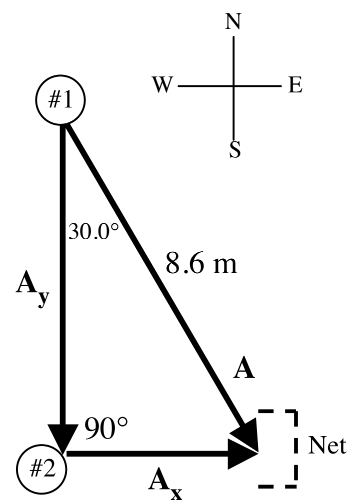
SOLUTION

a. The magnitude A_x of the displacement vector $\mathbf{A_x}$ is related to the length of the hypotenuse and the 30.0° angle by the sine function (Equation 1.1). The drawing shows that the direction of $\mathbf{A_x}$ is due east.

$$A_x = A\sin 30.0° = (8.6\text{ m})\sin 30.0° = \boxed{4.3\text{ m, due east}}$$

b. In a similar manner, the magnitude A_y of $\mathbf{A_y}$ can be found by using the cosine function (Equation 1.2). Its direction is due south.

$$A_y = A\cos 30.0° = (8.6\text{ m})\cos 30.0° = \boxed{7.4\text{ m, due south}}$$



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37. ***REASONING*** Using trigonometry, we can determine the angle θ from the relation $\tan\theta = A_y/A_x$:

SOLUTION

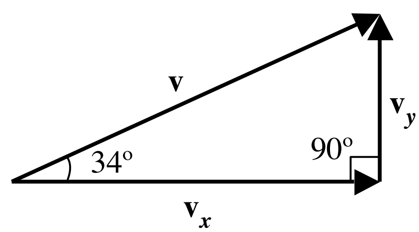
a. $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{12\text{ m}}{12\text{ m}}\right) = \boxed{45^\circ}$$

b. $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{12\text{ m}}{17\text{ m}}\right) = \boxed{35^\circ}$$

c. $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{17\text{ m}}{12\text{ m}}\right) = \boxed{55^\circ}$$

---

38. ***REASONING*** The drawing assumes that the horizontal direction along the ground is the *x* direction and shows the plane's velocity vector **v**, along with its horizontal component $\mathbf{v}_x$ and vertical component $\mathbf{v}_y$. These components, together with the velocity vector, form a right triangle, as indicated. Based on this right triangle, we can use the cosine function to determine the horizontal velocity component.



SOLUTION According to Equation 1.2, the cosine of an angle is the side of the right triangle adjacent to the angle divided by the hypotenuse. Thus, for the 34° angle in the drawing we have

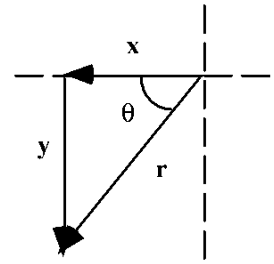
$$\cos 34^\circ = \frac{v_x}{v} \qquad \text{or} \qquad v_x = v\cos 34^\circ = (180\text{ m/s})\cos 34^\circ = \boxed{150\text{ m/s}}$$

---

39. $\boxed{\text{SSM}}$ ***REASONING*** The *x* and *y* components of **r** are mutually perpendicular; therefore, the magnitude of **r** can be found using the Pythagorean theorem. The direction of **r** can be found using the definition of the tangent function.

SOLUTION According to the Pythagorean theorem, we have

$$r = \sqrt{x^2 + y^2} = \sqrt{(-125\text{ m})^2 + (-184\text{ m})^2} = \boxed{222\text{ m}}$$



The angle θ is

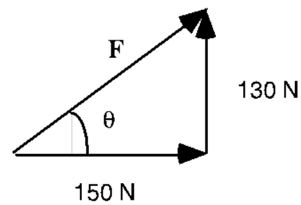
$$\theta = \tan^{-1}\left(\frac{184\text{ m}}{125\text{ m}}\right) = \boxed{55.8°}$$

---

40. ***REASONING AND SOLUTION***

a. From the Pythagorean theorem, we have

$$F = \sqrt{(150\text{ N})^2 + (130\text{ N})^2} = \boxed{2.0 \times 10^2\text{ N}}$$



b. The angle θ is given by

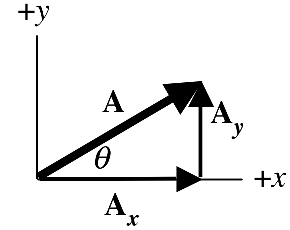
$$\theta = \tan^{-1}\left(\frac{130\text{ N}}{150\text{ N}}\right) = \boxed{41°}$$

---

41. ***REASONING*** Two vectors that are equal must have the same magnitude and direction. Equivalently, they must have identical x components and identical y components. We will begin by examining the given information with respect to these criteria, in order to see if there are obvious reasons why some of the vectors could not be equal. Then we will compare our choices for the equal vectors by calculating the magnitude and direction and the scalar components, as needed.

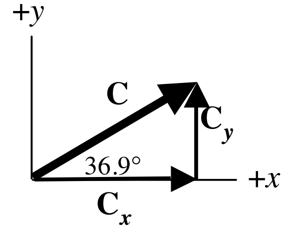
SOLUTION Vectors **A** and **B** cannot possibly be equal, because they have different x scalar components of $A_x = 80.0$ m and $B_x = 60.0$ m. Furthermore, vectors **B** and **C** cannot possibly be equal, because they have different magnitudes of $B = 75.0$ m and $C = 100.0$ m. Therefore, we conclude that $\boxed{\text{vectors } \mathbf{A} \text{ and } \mathbf{C} \text{ are the equal vectors}}$. To verify that this is indeed the case we have two choices. We can either calculate the magnitude and direction of **A** (and compare it to the given magnitude and direction of **C**) or determine the scalar components of **C** (and compare them to the given components of **A**). Either choice will do, although both are shown below.

The magnitude and direction of **A** are



$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(80.0\text{ m})^2 + (60.0\text{ m})^2} = 100.0\text{ m}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{60.0\text{ m}}{80.0\text{ m}}\right) = 36.9°$$



These results are identical to those given for **C**.

The scalar components of **C** are

$$C_x = C\cos 36.9° = (100.0\text{ m})\cos 36.9° = 80.0\text{ m}$$

$$C_y = C\sin 36.9° = (100.0\text{ m})\sin 36.9° = 60.0\text{ m}$$

These results are identical to the components given for **A**.

---

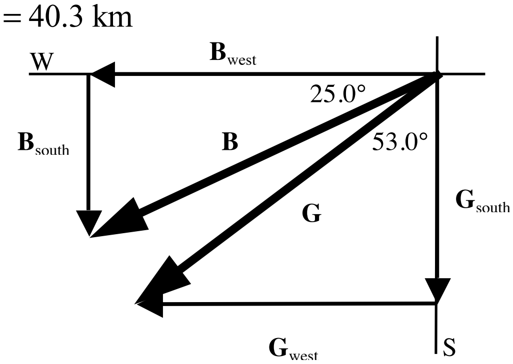
42. ***REASONING*** Because both boats travel at 101 km per hour, each one ends up (0.500 h)(101 km/h) = 50.5 km from the dock after a half-hour. They travel along straight paths, so this is the magnitude of both displacement vectors: $B = G = 50.5$ km. Since the displacement vector **G** makes an angle of 37° south of due west, its direction can also be expressed as 90° − 37° = 53° west of south. With these angles and the magnitudes of both vectors in hand, we can consider the westward and southward components of each vector.

SOLUTION

a. First we calculate the magnitudes of the westward component of the displacement of each boat and then subtract them to find the difference:

$$\text{Magnitude of } B_{\text{west}} = B\cos 25.0^{\text{o}} = (50.5\text{ km})\cos 25.0^{\text{o}} = 45.8\text{ km}$$

$$\text{Magnitude of } G_{\text{west}} = G\sin 53.0^{\text{o}} = (50.5\text{ km})\sin 53.0^{\text{o}} = 40.3\text{ km}$$



The blue boat travels farther by the following amount:

$$45.8\text{ km} - 40.3\text{ km} = \boxed{5.5\text{ km}}$$

b. Similarly, we find for the magnitudes of the southward components that

$$\text{Magnitude of } B_{\text{south}} = B\sin 25.0^{\text{o}} = (50.5\text{ km})\sin 25.0^{\text{o}} = 21.3\text{ km}$$

$$\text{Magnitude of } G_{\text{south}} = G\cos 53.0^{\text{o}} = (50.5\text{ km})\cos 53.0^{\text{o}} = 30.4\text{ km}$$

The green boat travels farther by the following amount:

$$30.4 \text{ km} - 21.3 \text{ km} = \boxed{9.1 \text{ km}}$$

---

43. SSM ***REASONING*** The force **F** and its two components form a right triangle. The hypotenuse is 82.3 newtons, and the side parallel to the $+x$ axis is F_x = 74.6 newtons. Therefore, we can use the trigonometric cosine and sine functions to determine the angle of **F** relative to the $+x$ axis and the component F_y of **F** along the $+y$ axis.

SOLUTION

a. The direction of **F** relative to the $+x$ axis is specified by the angle θ as

$$\theta = \cos^{-1}\left(\frac{74.6 \text{ newtons}}{82.3 \text{ newtons}}\right) = \boxed{25.0°} \qquad (1.5)$$

b. The component of **F** along the $+y$ axis is

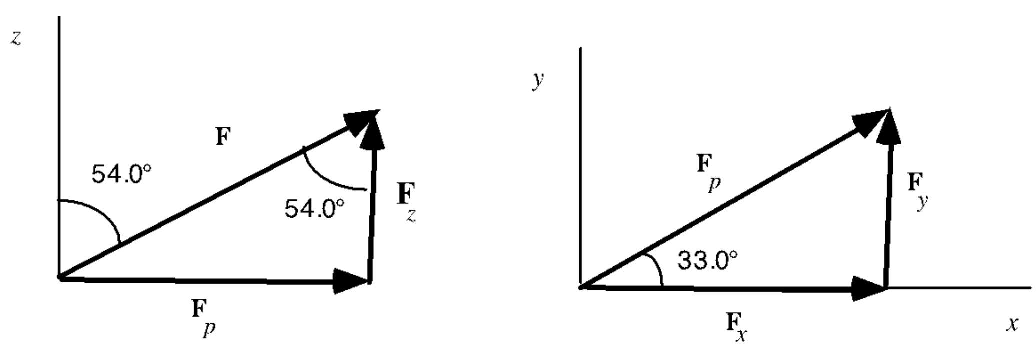
$$F_y = F \sin 25.0° = (82.3 \text{ newtons}) \sin 25.0° = \boxed{34.8 \text{ newtons}} \qquad (1.4)$$

---

44. ***REASONING AND SOLUTION*** The force **F** can be first resolved into two components; the z component F_z and the projection onto the x-y plane, F_p as shown on the left in the following figure. According to this figure,

$$F_p = F \sin 54.0° = (475 \text{ N}) \sin 54.0° = 384 \text{ N}$$

The projection onto the x-y plane, F_p, can then be resolved into x and y components.



a. From the figure on the right,

$$F_x = F_p \cos 33.0° = (384 \text{ N}) \cos 33.0° = \boxed{322 \text{ N}}$$

b. Also from the figure on the right,

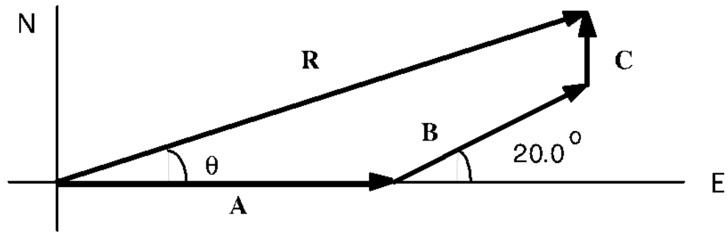
$$F_y = F_p \sin 33.0° = (384\text{ N}) \sin 33.0° = \boxed{209\text{ N}}$$

c. From the figure on the left,

$$F_z = F \cos 54.0° = (475\text{ N}) \cos 54.0° = \boxed{279\text{ N}}$$

---

45. SSM ***REASONING*** The individual displacements of the golf ball, **A**, **B**, and **C** are shown in the figure. Their resultant, **R,** is the displacement that would have been needed to "hole the ball" on the very first putt. We will use the component method to find **R**.



SOLUTION The components of each displacement vector are given in the table below.

Vector	***x* Components**	***y* Components**
A	(5.0 m) cos 0° = 5.0 m	(5.0 m) sin 0° = 0
B	(2.1 m) cos 20.0° = 2.0 m	(2.1 m) sin 20.0° = 0.72 m
C	(0.50 m) cos 90.0° = 0	(0.50 m) sin 90.0° = 0.50 m
R = A + B + C	7.0 m	1.22 m

The resultant vector **R** has magnitude

$$R = \sqrt{(7.0\text{ m})^2 + (1.22\text{ m})^2} = \boxed{7.1\text{ m}}$$

and the angle θ is

$$\theta = \tan^{-1}\left(\frac{1.22\text{ m}}{7.0\text{ m}}\right) = 9.9°$$

Thus, the required direction is $\boxed{9.9° \text{ north of east}}$.

---

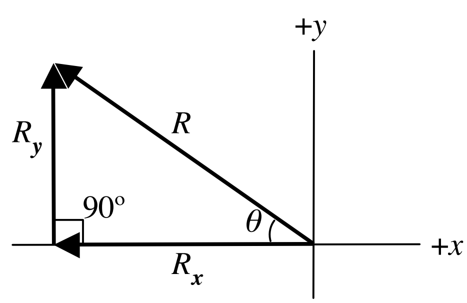
46. ***REASONING*** To apply the component method for vector addition, we must first determine the x and y components of each vector. The algebraic sum of the three x components gives the x component of the resultant. The algebraic sum of the three y components gives the y component of the resultant. Knowing the x and y components of the resultant will allow us to use the Pythagorean theorem to determine the magnitude of the resultant. Finally, the directional angle of the resultant will be obtained using the trigonometric sine function.

SOLUTION Referring to the drawing in the text, we see that the x and y components of the vectors are

$A_x = -A\cos 20.0° = -(5.00\text{ m})\cos 20.0° = -4.70\text{ m}$	$A_y = A\sin 20.0° = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$
$B_x = B\cos 60.0° = (5.00\text{ m})\cos 60.0° = 2.50\text{ m}$	$B_y = B\sin 60.0° = (5.00\text{ m})\sin 60.0° = 4.33\text{ m}$
$C_x = 0.00\text{ m}$	$C_y = -4.00\text{ m}$
$R_x = -4.70\text{ m} + 2.50\text{ m} + 0.00\text{ m} = -2.20\text{ m}$	$R_y = 1.71\text{ m} + 4.33\text{ m} - 4.00\text{ m} = 2.04\text{ m}$

Note that the value for A_x is negative because this component points in the $-x$ direction and that the value for C_x is zero because the vector **C** points along the $-y$ axis. Note also that the value for C_y is negative because the vector **C** points along the $-y$ axis.

The x component R_x of the resultant vector, being negative, points in the $-x$ direction. The y component R_y of the resultant vector, being positive, points in the $+y$ direction. The drawing shows these two components and the resultant vector. Since the components are perpendicular, the magnitude R of the resultant can be obtained using the Pythagorean theorem.



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20\text{ m})^2 + (2.04\text{ m})^2} = \boxed{3.00\text{ m}}$$

Referring to the drawing, we can see that $\sin\theta = R_x / R$, so that the directional angle θ is

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right) = \sin^{-1}\left(\frac{2.04\text{ m}}{3.00\text{ m}}\right) = 42.8°$$

Thus, the resultant vector points in a direction of $\boxed{42.8° \text{ above the negative } x \text{ axis}}$.

47. ***REASONING*** Using the component method for vector addition, we will find the x component of the resultant force vector by adding the x components of the individual vectors. Then we will find the y component of the resultant vector by adding the y components of the individual vectors. Once the x and y components of the resultant are known, we will use the Pythagorean theorem to find the magnitude of the resultant and trigonometry to find its direction. We will take east as the $+x$ direction and north as the $+y$ direction.

SOLUTION The x component of the resultant force **F** is

$$F_x = \underbrace{(2240\text{ N})\cos 34.0°}_{F_{Ax}} + \underbrace{(3160\text{ N})\cos 90.0°}_{F_{Bx}} = (2240\text{ N})\cos 34.0°$$

The y component of the resultant force **F** is

$$F_y = \underbrace{-(2240\text{ N})\sin 34.0°}_{F_{Ay}} + \underbrace{(-3160\text{ N})}_{F_{By}}$$

Using the Pythagorean theorem, we find that the magnitude of the resultant force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left[(2240\text{ N})\cos 34.0°\right]^2 + \left[-(2240\text{ N})\sin 34.0° - 3160\text{ N}\right]^2} = \boxed{4790\text{ N}}$$

Using trigonometry, we find that the direction of the resultant force is

$$\theta = \tan^{-1}\left[\frac{(2240\text{ N})\sin 34.0° + 3160\text{ N}}{(2240\text{ N})\cos 34.0°}\right] = \boxed{67.2° \text{ south of east}}$$

---

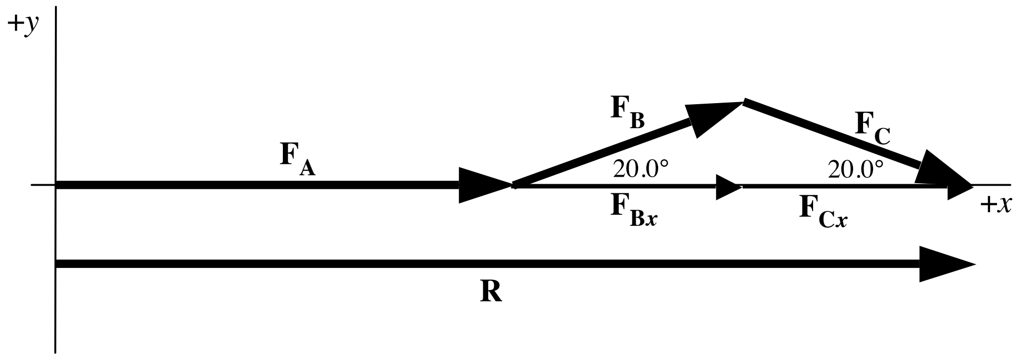
48. ***REASONING*** The resultant force in part *a* is $\mathbf{F_A}$, because that is the only force applied. The resultant force **R** in part *b*, where the two additional forces with identical magnitudes ($F_B = F_C = F$) are applied, is the sum of all three vectors: $\mathbf{R} = \mathbf{F_A} + \mathbf{F_B} + \mathbf{F_C}$. Because the magnitude R of the resultant force **R** is k times larger than the magnitude F_A of $\mathbf{F_A}$, we have

$$R = kF_A \qquad (1)$$

To solve the problem, we need to find an expression for R in terms of F_A and F. Let the $+x$ direction be in the direction of $\mathbf{F_A}$ and the $+y$ direction be upward (see the following drawing), and consider the resultant of the two additional force vectors in part *b*. Because $\mathbf{F_B}$ and $\mathbf{F_C}$ are directed symmetrically about the x axis, and have the same magnitude F, their y components are equal and opposite. Therefore, they cancel out of the vector sum **R**, leaving only the x components of $\mathbf{F_B}$ and $\mathbf{F_C}$. Now, since $\mathbf{F_A}$ also has only an x component, the resultant **R** can be written as the sum of $\mathbf{F_A}$ and the vector x components of $\mathbf{F_B}$ and $\mathbf{F_C}$:

$\mathbf{R} = \mathbf{F_A} + \mathbf{F_{Bx}} + \mathbf{F_{Cx}}$. Because these vectors all point in the same direction, we can write down a first expression for the magnitude of the resultant:

$$R = F_A + F_{Bx} + F_{Cx} \tag{2}$$



The x components of the vectors $\mathbf{F_B}$ and $\mathbf{F_C}$ are adjacent to the 20.0°-angles, and so are related to the common magnitude F of both vectors by Equation 1.2 $\left(\cos\theta = \frac{h_a}{h}\right)$, with $h_a = F_{Bx}$ and $h = F$:

$$F_{Bx} = F\cos\theta \qquad \text{and} \qquad F_{Cx} = F\cos\theta \tag{3}$$

SOLUTION First, we use Equations (3) to replace F_{Bx} and F_{Cx} in Equation (2):

$$R = F_A + F\cos\theta + F\cos\theta = F_A + 2F\cos\theta \tag{4}$$

Then, we combine Equation (4) with Equation (1) ($R = kF_A$) to eliminate R, and solve for the desired ratio F/F_A:

$$F_A + 2F\cos\theta = kF_A \quad \text{or} \quad 2F\cos\theta = kF_A - F_A \quad \text{or} \quad 2F\cos\theta = (k-1)F_A$$

$$\frac{F}{F_A} = \frac{k-1}{2\cos\theta} = \frac{2.00-1}{2\cos 20.0^\circ} = \boxed{0.532}$$

---

49. ***REASONING*** Using the component method, we find the components of the resultant **R** that are due east and due north. The magnitude and direction of the resultant **R** can be determined from its components, the Pythagorean theorem, and the tangent function.

SOLUTION The first four rows of the table below give the components of the vectors **A**, **B**, **C**, and **D**. Note that east and north have been taken as the positive directions; hence vectors pointing due west and due south will appear with a negative sign.

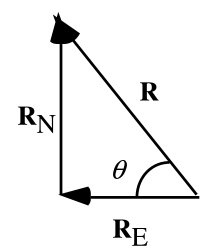
Vector	*East/West Component*	*North/South Component*
A	+ 2.00 km	0
B	0	+ 3.75 km
C	– 2.50 km	0
D	0	–3.00 km
R = A + B + C + D	– 0.50 km	+ 0.75 km

The fifth row in the table gives the components of **R**. The magnitude of **R** is given by the Pythagorean theorem as

$$R = \sqrt{(-\,0.50\text{ km})^2 + (+\,0.75\text{ km})^2} = \boxed{0.90\text{ km}}$$

The angle θ that **R** makes with the direction due west is

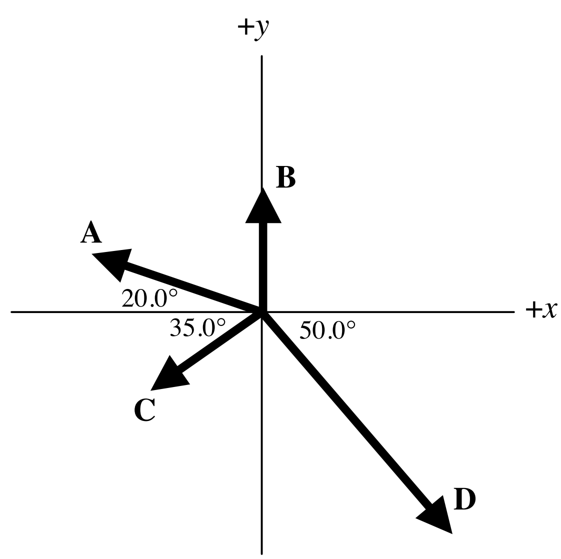
$$\theta = \tan^{-1}\left(\frac{0.75\text{ km}}{0.50\text{ km}}\right) = \boxed{56^\circ\text{ north of west}}$$



50. ***REASONING*** We will use the scalar x and y components of the resultant vector to obtain its magnitude and direction. To obtain the x component of the resultant we will add together the x components of each of the vectors. To obtain the y component of the resultant we will add together the y components of each of the vectors.

SOLUTION
The x and y components of the resultant vector **R** are R_x and R_y, respectively. In terms of these components, the magnitude R and the directional angle θ (with respect to the x axis) for the resultant are

$$R = \sqrt{R_x^2 + R_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \qquad (1)$$



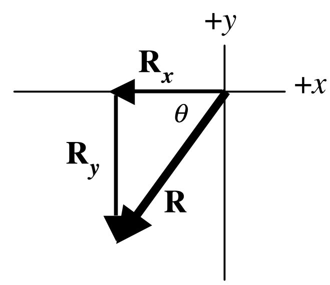
The following table summarizes the components of the individual vectors shown in the drawing:

Vector	x component	y component
A	$A_x = -(16.0\text{ m})\cos 20.0° = -15.0\text{ m}$	$A_y = (16.0\text{ m})\sin 20.0° = 5.47\text{ m}$
B	$B_x = 0\text{ m}$	$B_y = 11.0\text{ m}$
C	$C_x = -(12.0\text{ m})\cos 35.0° = -9.83\text{ m}$	$C_y = -(12.0\text{ m})\sin 35.0° = -6.88\text{ m}$
D	$D_x = (26.0\text{ m})\cos 50.0° = 16.7\text{ m}$	$D_y = -(26.0\text{ m})\sin 50.0° = -19.9\text{ m}$
R	$R_x = -15.0\text{ m} + 0\text{ m} - 9.83\text{ m} + 16.7\text{ m}$ $= -8.1\text{ m}$	$R_y = 5.47\text{ m} + 11.0\text{ m} - 6.88\text{ m} - 19.9\text{ m}$ $= -10.3\text{ m}$

Note that the component B_x is zero, because **B** points along the y axis. Note also that the components C_x and C_y are both negative, since **C** points between the $-x$ and $-y$ axes. Finally, note that the component D_y is negative since **D** points below the $+x$ axis.
Using equation (1), we find that

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-8.1\text{ m})^2 + (-10.3\text{ m})^2} = \boxed{13\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-10.3\text{ m}}{-8.1\text{ m}}\right) = \boxed{52°}$$



Since both R_x and R_y are negative, the resultant points between the $-x$ and $-y$ axes.

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51. ***REASONING*** If we let the directions due east and due north be the positive directions, then the desired displacement **A** has components

$$A_E = (4.8\text{ km})\cos 42° = 3.57\text{ km}$$

$$A_N = (4.8\text{ km})\sin 42° = 3.21\text{ km}$$

while the actual displacement **B** has components

$$B_E = (2.4\text{ km})\cos 22° = 2.23\text{ km}$$

$$B_N = (2.4\text{ km})\sin 22° = 0.90\text{ km}$$

Therefore, to reach the research station, the research team must go

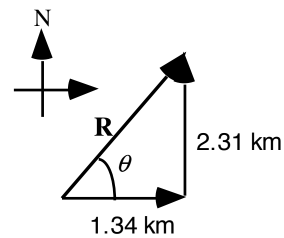
$$3.57\ \text{km} - 2.23\ \text{km} = 1.34\ \text{km, eastward}$$

and

$$3.21\ \text{km} - 0.90\ \text{km} = 2.31\ \text{km, northward}$$

SOLUTION

a. From the Pythagorean theorem, we find that the magnitude of the displacement vector required to bring the team to the research station is



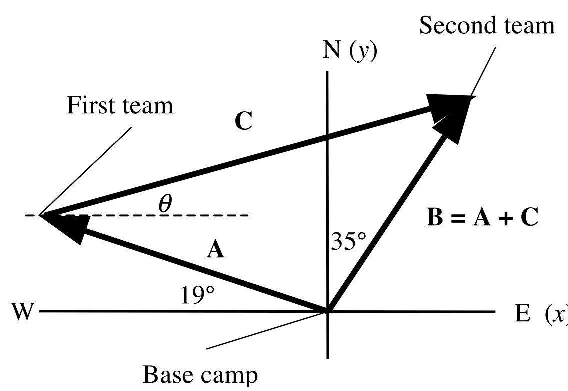
$$R = \sqrt{(1.34\ \text{km})^2 + (2.31\ \text{km})^2} = \boxed{2.7\ \text{km}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{2.31\ \text{km}}{1.34\ \text{km}}\right) = \boxed{6.0 \times 10^1\ \text{degrees, north of east}}$$

---

52. ***REASONING*** Let **A** be the vector from base camp to the first team, **B** the vector from base camp to the second team, and **C** the vector from the first team's position to the second team's position. **C** is the vector whose magnitude and direction are given by the first team's GPS unit. Since you can get from the base camp to the second team's position either by traveling along vector **B** alone, or by traveling first along **A** and then along **C**, we know that **B** is the vector sum of the other two: **B** = **A** + **C**.



The reading on the first team's GPS unit is then **C** = **B** − **A**. The components of **C** are found from the components of **A** and **B**: $C_x = B_x - A_x$, $C_y = B_y - A_y$. Once we have these components, we can calculate the magnitude and direction of **C**, as shown on the GPS readout. Because the first team is northwest of camp, and the second team is northeast, we expect the vector **C** to be directed east and either north or south.

SOLUTION Let east serve as the positive x direction and north as the positive y direction. We then calculate the components of **C**, noting that **B**'s components are both positive, and **A**'s x-component is negative:

$$C_x = \underbrace{(29\text{ km})\sin 35^\circ}_{B_x} - \underbrace{(-38\text{ km})\cos 19^\circ}_{A_x} = 53\text{ km}$$

$$C_y = \underbrace{(29\text{ km})\cos 35^\circ}_{B_y} - \underbrace{(38\text{ km})\sin 19^\circ}_{A_y} = 11\text{ km}$$

The second team is therefore 53 km east of the first team (since C_x is positive), and 11 km north (since C_y is positive). The straight-line distance between the teams can be calculated with the Pythagorean theorem (Equation 1.7):

$$C = \sqrt{C_x^{\,2} + C_y^{\,2}} = \sqrt{(53\text{ km})^2 + (11\text{ km})^2} = \boxed{54\text{ km}}$$

The direction of the vector **C** is to be measured relative to due east, so we apply the inverse tangent function (Equation 1.6) to get the angle θ:

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{11\text{ km}}{53\text{ km}}\right) = \boxed{12^\circ}$$

---

53. $\boxed{\text{SSM}}$ ***REASONING*** Since the finish line is coincident with the starting line, the net displacement of the sailboat is zero. Hence the sum of the components of the displacement vectors of the individual legs must be zero. In the drawing in the text, the directions to the right and upward are taken as positive.

SOLUTION In the horizontal direction $R_h = A_h + B_h + C_h + D_h = 0$

$$R_h = (3.20\text{ km})\cos 40.0^\circ - (5.10\text{ km})\cos 35.0^\circ - (4.80\text{ km})\cos 23.0^\circ + D\cos\theta = 0$$

$$D\cos\theta = 6.14\text{ km} \qquad (1)$$

In the vertical direction $R_v = A_v + B_v + C_v + D_v = 0$.

$$R_v = (3.20\text{ km})\sin 40.0^\circ + (5.10\text{ km})\sin 35.0^\circ - (4.80\text{ km})\sin 23.0^\circ - D\sin\theta = 0.$$

$$D\sin\theta = 3.11\text{ km} \qquad (2)$$

Dividing (2) by (1) gives

$$\tan\theta = (3.11\text{ km})/(6.14\text{ km}) \quad \text{or} \quad \theta = \boxed{26.9^\circ}$$

Solving (1) gives

$$D = (6.14 \text{ km})/\cos 26.9° = \boxed{6.88 \text{ km}}$$

---

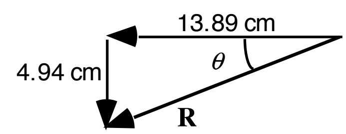
54. ***REASONING*** The following table shows the components of the individual displacements and the components of the resultant. The directions due east and due north are taken as the positive directions.

Displacement	***East/West Component***	***North/South Component***
(1)	–27.0 cm	0
(2)	–(23.0 cm) cos 35.0° = –18.84 cm	–(23.0 cm) sin 35.0° = –13.19 cm
(3)	(28.0 cm) cos 55.0° = 16.06 cm	–(28.0 cm) sin 55.0° = –22.94 cm
(4)	(35.0 cm) cos 63.0° = 15.89 cm	(35.0 cm) sin 63.0° = 31.19 cm
Resultant	–13.89 cm	–4.94 cm

SOLUTION

a. From the Pythagorean theorem, we find that the magnitude of the resultant displacement vector is

$$R = \sqrt{(13.89 \text{ cm})^2 + (4.94 \text{ cm})^2} = \boxed{14.7 \text{ cm}}$$

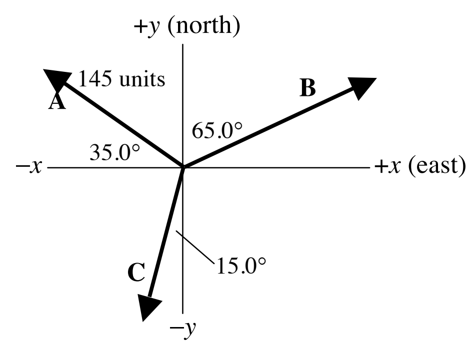


b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{4.94 \text{ cm}}{13.89 \text{ cm}}\right) = \boxed{19.6°, \text{ south of west}}$$

---

55. ***REASONING*** The drawing shows the vectors **A**, **B**, and **C**. Since these vectors add to give a resultant that is zero, we can write that **A** + **B** + **C** = 0. This addition will be carried out by the component method. This means that the *x*-component of this equation must be zero ($A_x + B_x + C_x = 0$) and the *y*-component must be zero ($A_y + B_y + C_y = 0$). These two equations will allow us to find the magnitudes of **B** and **C**.



SOLUTION The *x*- and *y*-components of **A**, **B**, and **C** are given in the table below. The plus and minus signs indicate whether the components point along the positive or negative axes.

Vector	***x Component***	***y Component***

A	–(145 units) cos 35.0° = –119 units	+(145 units) sin 35.0° = +83.2 units
B	+*B* sin 65.0° = +*B* (0.906)	+*B* cos 65.0° = +*B* (0.423)
C	–*C* sin 15.0° = –*C* (0.259)	–*C* cos 15.0° = –*C* (0.966)
A + B + C	–119 units + *B* (0.906) – *C* (0.259)	+83.2 units + *B* (0.423) – *C* (0.966)

Setting the separate *x*- and *y*- components of **A + B + C** equal to zero gives

x-component $(-119 \text{ units}) + B\,(0.906) - C\,(0.259) = 0$

y-component $(+83.2 \text{ units}) + B\,(0.423) - C\,(0.966) = 0$

Solving these two equations simultaneously, we find that

a. $B = \boxed{178 \text{ units}}$ b. $C = \boxed{164 \text{ units}}$

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56. ***REASONING*** We know that the three displacement vectors have a resultant of zero, so that **A + B + C = 0**. This means that the sum of the *x* components of the vectors and the sum of the *y* components of the vectors are separately equal to zero. From these two equations we will be able to determine the magnitudes of vectors **B** and **C**. The directions east and north are, respectively, the +*x* and +*y* directions.

SOLUTION Setting the sum of the *x* components of the vectors and the sum of the *y* components of the vectors separately equal to zero, we have

$$\underbrace{(1550 \text{ m})\cos 25.0^\circ}_{A_x} + \underbrace{B \sin 41.0^\circ}_{B_x} + \underbrace{(-C \cos 35.0^\circ)}_{C_x} = 0$$

$$\underbrace{(1550 \text{ m})\sin 25.0^\circ}_{A_y} + \underbrace{(-B \cos 41.0^\circ)}_{B_y} + \underbrace{C \sin 35.0^\circ}_{C_y} = 0$$

These two equations contain two unknown variables, *B* and *C*. They can be solved simultaneously to show that

a. $B = \boxed{5550 \text{ m}}$ and b. $C = \boxed{6160 \text{ m}}$

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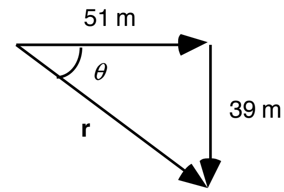
57. ***REASONING*** The shortest distance between the tree and the termite mound is equal to the magnitude of the chimpanzee's displacement **r**.

SOLUTION

a. From the Pythagorean theorem, we have



$$r = \sqrt{(51\text{ m})^2 + (39\text{ m})^2} = \boxed{64\text{ m}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{39\text{ m}}{51\text{ m}}\right) = \boxed{37°\text{ south of east}}$$

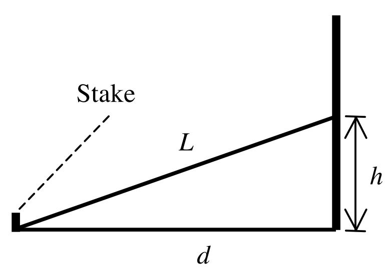
---

58. ***REASONING*** When the monkey has climbed as far up the pole as it can, its leash is taut, making a straight line from the stake to the monkey, that is, $L = 3.40$ m long. The leash is the hypotenuse of a right triangle, and the other sides are a line drawn from the stake to the base of the pole ($d = 3.00$ m), and a line from the base of the pole to the monkey (height = h).

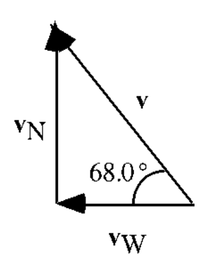
SOLUTION These three lengths are related by the Pythagorean theorem (Equation 1.7):

$$h^2 + d^2 = L^2 \qquad \text{or} \qquad h^2 = L^2 - d^2$$

$$h = \sqrt{L^2 - d^2} = \sqrt{(3.40\text{ m})^2 - (3.00\text{ m})^2} = \boxed{1.6\text{ m}}$$



59. SSM ***REASONING*** The ostrich's velocity vector **v** and the desired components are shown in the figure at the right. The components of the velocity in the directions due west and due north are $\mathbf{v}_W$ and $\mathbf{v}_N$, respectively. The sine and cosine functions can be used to find the components.



SOLUTION

a. According to the definition of the sine function, we have for the vectors in the figure

$$\sin\,\theta = \frac{v_N}{v} \quad \text{or} \quad v_N = v\sin\,\theta = (17.0\text{ m/s})\sin 68° = \boxed{15.8\text{ m/s}}$$

b. Similarly,

$$\cos\,\theta = \frac{v_W}{v} \quad \text{or} \quad v_W = v\cos\,\theta = (17.0\text{ m/s})\cos 68.0° = \boxed{6.37\text{ m/s}}$$

---

60. ***REASONING*** In the expression for the volume flow rate, the dimensions on the left side of the equals sign are $[L]^3/[T]$. If the expression is to be valid, the dimensions on the right side of the equals sign must also be $[L]^3/[T]$. Thus, the dimensions for the various symbols on the right must combine algebraically to yield $[L]^3/[T]$. We will substitute the dimensions for each symbol in the expression and treat the dimensions of [M], [L], and [T] as algebraic variables, solving the resulting equation for the value of the exponent n.

SOLUTION We begin by noting that the symbol π and the number 8 have no dimensions. It follows, then, that

$$Q = \frac{\pi R^n\left(P_2 - P_1\right)}{8\eta L} \quad \text{or} \quad \frac{[L]^3}{[T]} = \frac{[L]^n \dfrac{\cancel{[M]}}{[L][T]^2}}{\dfrac{\cancel{[M]}}{\cancel{[L]}[T]}\cancel{[L]}} = \frac{[L]^n[T]}{[L][T]^2} = \frac{[L]^n}{[L][T]}$$

$$\frac{[L]^3}{\cancel{[T]}} = \frac{[L]^n}{[L]\cancel{[T]}} \quad \text{or} \quad [L]^3 = \frac{[L]^n}{[L]} \quad \text{or} \quad [L]^3[L] = [L]^4 = [L]^n$$

Thus, we find that $\boxed{n = 4}$.

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61. ***REASONING AND SOLUTION*** The east and north components are, respectively

a. $A_e = A\cos\theta = (155\text{ km})\cos 18.0° = \boxed{147\text{ km}}$

b. $A_n = A\sin\theta = (155\text{ km})\sin 18.0° = \boxed{47.9\text{ km}}$

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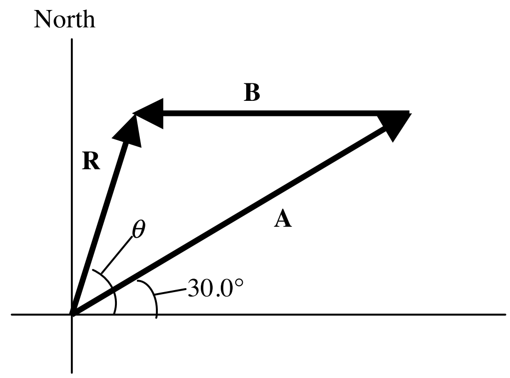
62. ***REASONING*** According to the component method for vector addition, the x component of the resultant vector is the sum of the x component of **A** and the x component of **B**. Similarly, the y component of the resultant vector is the sum of the y component of **A** and the y component of **B**. The magnitude R of the resultant can be obtained from the x and y components of the resultant by using the Pythagorean theorem. The directional angle θ can be obtained using trigonometry.



SOLUTION We find the following results:

$$R_x = \underbrace{(244\text{ km})\cos 30.0^\circ}_{A_x} + \underbrace{(-175\text{ km})}_{B_x} = 36\text{ km}$$

$$R_y = \underbrace{(244\text{ km})\sin 30.0^\circ}_{A_y} + \underbrace{(0\text{ km})}_{B_y} = 122\text{ km}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(36\text{ km})^2 + (122\text{ km})^2} = \boxed{127\text{ km}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{122\text{ km}}{36\text{ km}}\right) = \boxed{74^\circ}$$

____

63. $\boxed{\textbf{SSM}}$ ***REASONING*** The performer walks out on the wire a distance d, and the vertical distance to the net is h. Since these two distances are perpendicular, the magnitude of the displacement is given by the Pythagorean theorem as $s = \sqrt{d^2 + h^2}$. Values for s and h are given, so we can solve this expression for the distance d. The angle that the performer's displacement makes below the horizontal can be found using trigonometry.

SOLUTION

a. Using the Pythagorean theorem, we find that

$$s = \sqrt{d^2 + h^2} \quad \text{or} \quad d = \sqrt{s^2 - h^2} = \sqrt{(26.7\text{ ft})^2 - (25.0\text{ ft})^2} = \boxed{9.4\text{ ft}}$$

b. The angle θ that the performer's displacement makes below the horizontal is given by

$$\tan\theta = \frac{h}{d} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{h}{d}\right) = \tan^{-1}\left(\frac{25.0 \text{ ft}}{9.4 \text{ ft}}\right) = \boxed{69°}$$

---

64. ***REASONING*** The force vector **F** points at an angle of θ above the +x axis. Therefore, its x and y components are given by $F_x = F \cos\theta$ and $F_y = F \sin\theta$.

SOLUTION a. The magnitude of the vector can be obtained from the y component as follows:

$$F_y = F\sin\theta \quad \text{or} \quad F = \frac{F_y}{\sin\theta} = \frac{290 \text{ N}}{\sin 52°} = \boxed{370 \text{ N}}$$

b. Now that the magnitude of the vector is known, the x component of the vector can be calculated as follows:

$$F_x = F\cos\theta = (370 \text{ N})\cos 52° = \boxed{+230 \text{ N}}$$

---

65. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** We take due north to be the direction of the +y axis. Vectors **A** and **B** are the components of the resultant, **C**. The angle that **C** makes with the x axis is then $\theta = \tan^{-1}(B/A)$. The symbol u denotes the units of the vectors.

a. Solving for B gives

$$B = A \tan\theta = (6.00 \text{ u}) \tan 60.0° = \boxed{10.4 \text{ u}}$$

b. The magnitude of **C** is

$$C = \sqrt{A^2 + B^2} = \sqrt{(6.00 \text{ u})^2 + (10.4 \text{ u})^2} = \boxed{12.0 \text{ u}}$$

–

66. ***REASONING*** We are given that the vector sum of the three forces is zero, so $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = 0$ N. Since $\mathbf{F_1}$ and $\mathbf{F_2}$ are known, $\mathbf{F_3}$ can be found from the relation $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$. We will use the x- and y-components of this equation to find the magnitude and direction of $\mathbf{F_3}$.

SOLUTION The x- and y-components of the equation $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$ are:

x-component $$F_{3x} = -\left(F_{1x} + F_{2x}\right) \quad (1)$$

y-component $$F_{3y} = -\left(F_{1y} + F_{2y}\right) \quad (2)$$

The table below gives the x- and y-components of $\mathbf{F_1}$ and $\mathbf{F_2}$:

Vector	x component	y component
$\mathbf{F_1}$	$F_{1x} = -(21.0 \text{ N}) \sin 30.0° = -10.5 \text{ N}$	$F_{1y} = -(21.0 \text{ N}) \cos 30.0° = +18.2 \text{ N}$
$\mathbf{F_2}$	$F_{2x} = +15.0 \text{ N}$	$F_{2y} = 0 \text{ N}$

Substituting the values for F_{1x} and F_{2x} into Equation (1) gives

$$F_{3x} = -\left(F_{1x} + F_{2x}\right) = -\left(-10.5 \text{ N} + 15.2 \text{ N}\right) = -4.5 \text{ N}$$

Substituting F_{1y} and F_{2y} into Equation (2) gives

$$F_{3y} = -\left(F_{1y} + F_{2y}\right) = -\left(+18.2 \text{ N} + 0 \text{ N}\right) = -18.2 \text{ N}$$

The magnitude of $\mathbf{F_3}$ can now be obtained by employing the Pythagorean theorem:

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(-4.5 \text{ N})^2 + (-18.2 \text{ N})^2} = \boxed{18.7 \text{ N}}$$

The angle θ that $\mathbf{F_3}$ makes with respect to the $-x$ axis can be determined from the inverse tangent function (Equation 1.6),

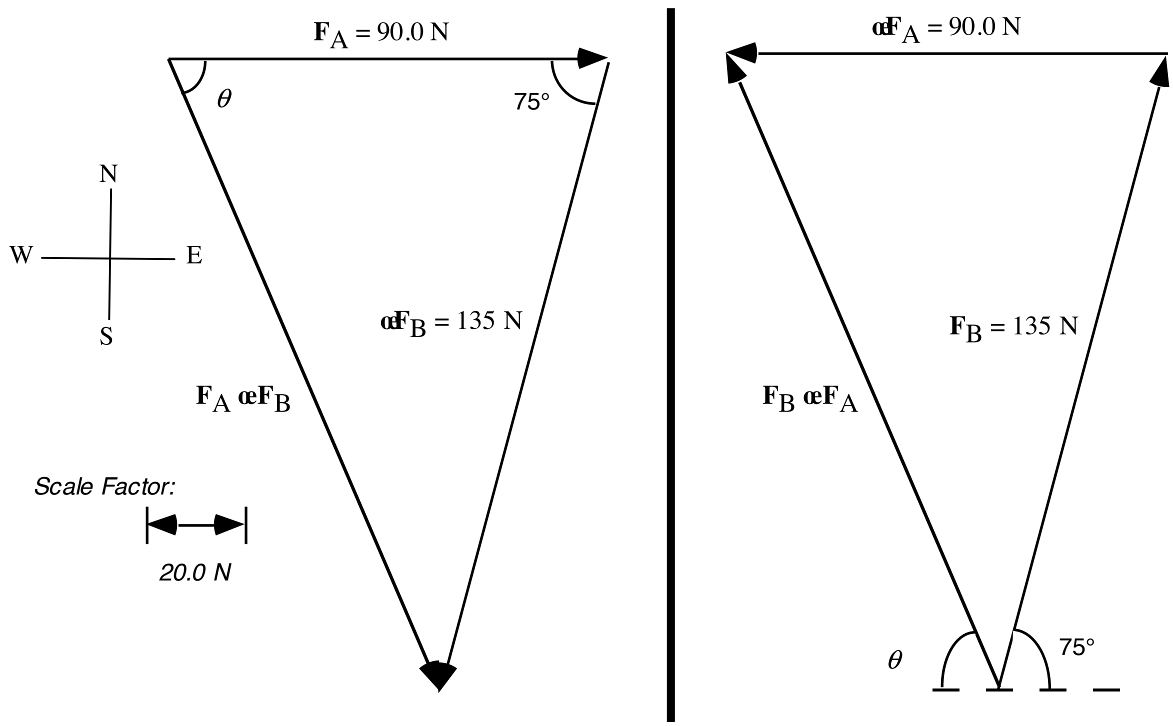
$$\theta = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left(\frac{-18.2 \text{ N}}{-4.5 \text{ N}}\right) = \boxed{76°}$$

–

67. ***REASONING AND SOLUTION*** The following figures are scale diagrams of the forces drawn tail-to-head. The scale factor is shown in the figure.

a. From the figure on the left, we see that $\boxed{F_\text{A} - F_\text{B} = 142 \text{ N}, \ \theta = 67° \text{ south of east}}$.

b. Similarly, from the figure on the right, $\boxed{F_\text{B} - F_\text{A} = 142 \text{ N}, \ \theta = 67° \text{ north of west}}$.



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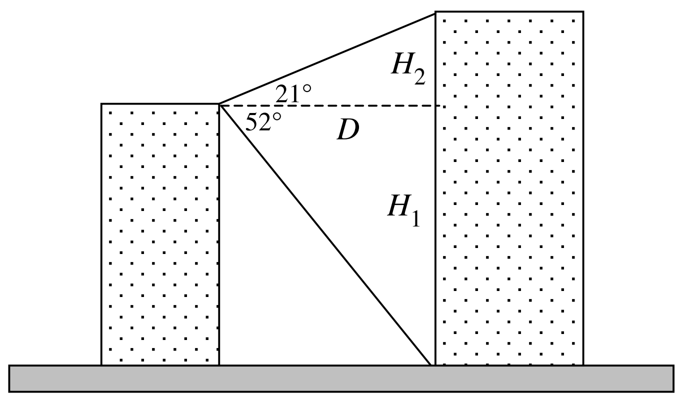
68. ***REASONING*** There are two right triangles in the drawing. Each contains the common side that is shown as a dashed line and is labeled D, which is the distance between the buildings. The hypotenuse of each triangle is one of the lines of sight to the top and base of the taller building. The remaining (vertical) sides of the triangles are labeled H_1 and H_2.



Since the height of the taller building is $H_1 + H_2$ and the height of the shorter building is H_1, the ratio that we seek is $(H_1 + H_2)/H_1$. We will use the tangent function to express H_1 in terms of the 52° angle and to express H_2 in terms of the 21° angle. The unknown distance D will be eliminated algebraically when the ratio $(H_1 + H_2)/H_1$ is calculated.

SOLUTION The ratio of the building heights is

$$\frac{\text{Height of taller building}}{\text{Height of shorter building}} = \frac{H_1 + H_2}{H_1}$$

Using the tangent function, we have that

$$\tan 52° = \frac{H_1}{D} \quad \text{or} \quad H_1 = D \tan 52°$$

$$\tan 21° = \frac{H_2}{D} \quad \text{or} \quad H_2 = D \tan 21°$$

Substituting these results into the expression for the ratio of the heights gives

$$\frac{\text{Height of taller building}}{\text{Height of shorter building}} = \frac{H_1 + H_2}{H_1} = \frac{D\tan 52° + D\tan 21°}{D\tan 52°}$$

$$= 1 + \frac{\tan 21°}{\tan 52°} = \boxed{1.30}$$

Since 1.30 is less than 1.50, $\boxed{\text{your friend is wrong}}$.

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69. ***REASONING AND SOLUTION*** If **D** is the unknown vector, then **A** + **B** + **C** + **D** = 0 requires that $D_E = -(A_E + B_E + C_E)$ or

$$D_E = (113\text{ u}) \cos 60.0° - (222\text{ u}) \cos 35.0° - (177\text{ u}) \cos 23.0° = \boxed{-288\text{ units}}$$

The minus sign indicates that D_E has a direction of due west.

Also, $D_N = -(A_N + B_N + C_N)$ or

$$D_N = (113\text{ u}) \sin 60.0° + (222\text{ u}) \sin 35.0° - (177\text{ u}) \sin 23.0° = \boxed{156\text{ units}}$$

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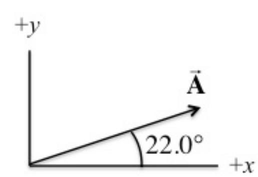
70. ***CONCEPTS***

(i) When two vectors are equal, each has the same magnitude and each has the same direction.

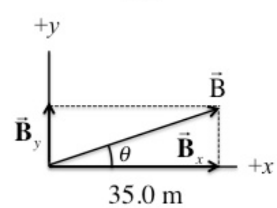
(ii) When two vectors are equal, the *x* component of vector $\vec{\mathbf{A}}$ equals the *x* component of vector $\vec{\mathbf{B}}$ (A_x=B_x), and the *y* component of vector $\vec{\mathbf{A}}$ equals the *y* component of vector $\vec{\mathbf{B}}$ (A_y=B_y).



(*a*)

CALCULATIONS

We focus on the fact that the *x* components of the vectors are equal and the *y* components of the vectors are equal. Referring to the figure, we write:



(*b*)

$$A\cos 22.0° = 35.0\text{ m} \quad (1)$$

$$A\sin 22.0° = B_y \quad (2)$$

Dividing equation (2) by equation (1) gives:

$$\frac{A\sin 22.0°}{A\cos 22.0°} = \frac{B_y}{35.0\text{ m}}$$

Therefore,

$$B_y = (35.0\text{ m})\tan 22.0° = \boxed{14.1\text{ m}}$$

Solving eq. (1) directly for A gives

$$A = \frac{35.0\text{ m}}{\cos 22.0°} = \boxed{37.7\text{ m}}$$

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71. SSM ***CONCEPTS*** **(i)** The magnitude of $\vec{\mathbf{A}}$ is given by the Pythagorean theorem in the form $A = \sqrt{A_x^2 + A_y^2}$, since a vector and its components form a right triangle.

(ii) Yes. The vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{R}}$ each have a zero value for their y component. This is because these vectors are parallel to the x axis.

(iii) The fact that $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = \vec{\mathbf{R}}$ means that the sum of the x components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ equals the x component of $\vec{\mathbf{R}}$: $A_x + B_x + C_x = R_x$. A similar relation holds for the y components: $A_y + B_y + C_y = R_y$.

CALCULATIONS We will use the Pythagorean theorem to relate A to the components of $\vec{\mathbf{A}}$: $A = \sqrt{A_x^2 + A_y^2}$.

To obtain the value for A_x we use the fact that the sum of the components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ equals the x component of $\vec{\mathbf{R}}$:

$$A_x + \underbrace{10.0\text{ m}}_{B_x} + \underbrace{(23.0\text{ m})\cos 50°}_{C_x} = \underbrace{35.0\text{ m}}_{R_x} \qquad \text{or } A_x = 10.2\text{ m}$$

Similarly, for the y components we have

$$A_y + \underbrace{0\text{ m}}_{B_y} + \underbrace{\left[-(23.0\text{ m})\sin 50°\right]}_{C_y} = \underbrace{0\text{ m}}_{R_y} \qquad \text{or} \quad A_y = 17.6\text{ m}$$

Using these values for A_x and A_y, we find that the magnitude of $\vec{\mathbf{A}}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(10.2\ \text{m})^2 + (17.6\ \text{m})^2} = \boxed{20.3\ \text{m}}$$

We also use the components A_x and A_y to find the directional angle θ:

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{17.6\ \text{m}}{10.2\ \text{m}}\right) = \boxed{59.9^\circ}$$